The Production Function for Housing: Evidence from France

Pierre-Philippe Combes* †
University of Lyon and Sciences Po

Gilles Duranton*‡
University of Pennsylvania

Laurent Gobillon*§
Paris School of Economics

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Abstract: We propose a new non-parametric approach to estimate the production function for housing. Our estimation treats output as a latent variable and relies on the first-order condition for profit maximisation with respect to non-land inputs by competitive house builders. For parcels of a given size, we compute housing by summing across the marginal products of non-land inputs. Differences in non-land inputs are caused by differences in land prices that reflect differences in the demand for housing across locations. We implement our methodology on newly-built single-family homes in France. We find that the production function for housing is reasonably well, though not perfectly, approximated by a Cobb-Douglas function and close to constant returns. After correcting for differences in user costs between land and non-land inputs and taking care of some estimation concerns, we estimate an elasticity of housing production with respect to non-land inputs of about 0.80.

Key words: housing, production function.

JEL classification: R14, R31, R32

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†Univ Lyon, CNRS, GATE-LSE UMR 5824, 93 Chemin des Mouilles, 69131 Ecully, France and Sciences Po, Economics Department, 28, Rue des Saints-Pères, 75007 Paris, France (e-mail: `ppcombes@gmail.com`; website: `https://www.gate.cnrs.fr/ppcombes/`). Also affiliated with the Centre for Economic Policy Research.

‡Wharton School, University of Pennsylvania, 3620 Locust Walk, Philadelphia, PA 19104, USA (e-mail: `duranton@wharton.upenn.edu`; website: `https://real-estate.wharton.upenn.edu/profile/21470/`). Also affiliated with the Center for Economic Policy Research, the Spatial Economic Centre at the London School of Economics, and the Rimini Centre for Economic Analysis.

§PSE-CNRS, 48 Boulevard Jourdan, 75014 Paris, France (e-mail: `laurent.gobillon@psemail.eu`; website: `http://laurent.gobillon.free.fr/`). Also affiliated with the Centre for Economic Policy Research and the Institute for the Study of Labor (IZA).

1. Introduction

We propose a new non-parametric approach to estimate the production function for housing. Our estimation treats output as a latent variable and relies on the first-order condition for profit maximisation by competitive house builders. For parcels of a given size, we compute housing by summing across the marginal products of non-land inputs. In turn, differences in non-land inputs are caused by differences in land prices that reflect differences in the demand for housing across locations. We implement our methodology on recently-built single-family homes in France. We find that the production function for housing is reasonably well, though not perfectly, approximated by a Cobb-Douglas function and close to constant returns. differences in user costs between land and non-land inputs and taking care of some estimation concerns, we estimate an elasticity of housing production with respect to non-land inputs of about 0.80.

A good understanding of the supply of housing is important for a number of reasons. First, housing is an unusually important good. It arguably provides an essential service to households and represents more than 30% of their expenditure in both the US and France.[1] It is also an important asset. The value of the US residential stock owned by households was around 20 trillion dollar in 2007 (Gyourko, 2009). French households owned about 4.6 trillion dollar worth of housing in 2011 (Mauro, 2013). For both countries, this represents about 180% of their gross domestic product.

Housing and the construction industry also matter to the broader economy. The construction industry is arguably an important driver of the business cycle (e.g., Davis and Heathcote, 2005). The role of housing in the great recession has been studied by, among others, Chatterjee and Eyigungor (2015) and Kiyotaki, Michaelides, and Nikolov (2011). The broader effects of housing are not limited to the business cycle. Housing has also been argued to affect a variety of aggregate variables such as unemployment (Head and Lloyd-Ellis, 2012, Rupert and Wasmer, 2012) or economic growth (Davis, Fisher, and Whited, 2014, Hsieh and Moretti, 2015).

Finally, and most importantly, housing is also central to our understanding of cities. Different locations within a city offer different levels of employment and shopping accessibility and bundles of amenities. Housing production is central in transforming the demand for locations from households into patterns of land use and housing consumption. Unsurprisingly, housing is at the heart of land use models in the spirit of Alonso (1964), Muth (1969), and Mills (1967) that form the core of modern urban economics. Related to this, the welfare consequences of land use regulations

[1] See Combes, Duranton, and Gobillon (2016) for sources and further discussion of the evidence.

depend on the shape of the housing production function (Larson and Yezer, 2015). For instance, the consequences of minimum lot size requirements will depend on how easily substitutable land is in the production of housing.

Following Muth's (1969, 1975) pioneering efforts, there is a long tradition of work that estimates a production function for housing. Some of this work mirrors standard practice in productivity studies and regresses a measure of housing output on land and other inputs. When we observe the price of a transaction for a house, it is hard to separate between the price of housing per unit and the quality-adjusted amount of housing that this house offers. Then, a regression of "housing" on land and non-land inputs is likely to contain the unit price of housing in its error term. Since we expect this price to determine non-land inputs, the regression will not appropriately identify the production function for housing. This is a version of the unobserved price / unobserved quality problem that usually plagues the estimation of production functions.[2]

A popular alternative is to estimate the elasticity of substitution between land and other inputs directly by regressing the ratio of land to non-land inputs on the unit price of land. Because the price of land is usually inferred from the value of a house minus the replacement cost of non-land inputs, this regression suffers from reverse causation. With these caveats in mind, extant results are generally supportive of constant returns to scale in the production of housing and estimates for the elasticity of substitution between land and other inputs typically range between 0.50 and 0.75.[3]

To summarise, housing is highly heterogeneous and land, an immobile factor whose price is often hard to observe, plays a particularly important role in its production. These features call for specific estimation techniques, impose strong data constraints, and require careful attention to the sources of variation used for identification.

To meet our first challenge and separate the quantity of housing from its price per unit, we develop a novel estimation approach, which relies on three main assumptions. First we assume a production function for housing, which uses land and non-land inputs as primary factors.[4] Since it cannot be directly observed, the quantity of housing is best thought of as a latent variable. Second,

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[2]See Ackerberg, Benkard, Berry, and Pakes (2007) and Syverson (2011) for discussion of the issues associated with the estimation of production functions.

[3]Thorsnes (1997) is an interesting exception. He estimates an elasticity of substitution between land and other inputs statistically undistinguishable from one using high-quality data for which he observes both the price of land prior to construction and the price of the house when it is sold.

[4]The notion of a production for housing services can be traced back to Muth (1960) and Olsen (1969).

house builders maximise profit. They choose how much non-land inputs to use in order to build a house on a particular parcel of land given the price that households are willing to pay for each unit of housing on this parcel. Third, we assume free entry among builders.

The first-order condition for profit maximisation by house builders implies that the marginal value product of non-land inputs should be equal to their user cost. Then, under free entry, the difference between the price of a house and the cost of the non-land inputs used to produce it should be equal to the price of the parcel. We can use this condition to eliminate the price of housing from the first-order condition and obtain a partial differential equation where the marginal product of non-land inputs depends on the quantity of housing produced and the cost and quantity of both factors.[5,6] Given parcel size, this partial differential equation can be solved to obtain a non-parametric estimate of the amount of housing as a function of non-land inputs. Because our estimation is conditional on parcel size, the production function for housing is only partially identified.

The second challenge is to find appropriate data. Our methodology requires information about the price of parcels, their size, and the cost of construction. The unique data we use satisfy these requirements. They consist of several large annual cross-sections of land parcels sold in France with a building permit for a single-family home and the cost of building this home.

Given our approach and the data at hand, the third challenge is to use an appropriate source of variation. Although our estimation technique is non-standard, it remains that the supply of housing can only be identified from variation in the demand for housing across parcels, not from unobserved differences in supply conditions. We develop an instrumental variable approach that relies on systematic determinants of the demand for housing, namely the urban area of a parcel and its location within this urban area. Housing located closer to the centre of Paris is more expensive than housing located further away in the suburbs. This is more plausibly caused by differences

[5]Gandhi, Navarro, and Rivers (2013) jointly use the first-order condition for profit maximisation and the production function to eliminate unobserved persistent firm heterogeneity in productivity. This leads them to derive a partial differential equation similar to ours. For partial identification of the production function of housing, we only rely on the integration of this differential equation. For full identification, we make further assumptions about returns to scale in production. By contrast, Gandhi *et al.* (2013) make assumptions about the dynamics of productivity, insert the related equation into the production function and estimate the resulting specification that includes both the current and lagged values of inputs.

[6]Our approach consists in eliminating the unobserved price of output and rely about information on input prices and quantities. An alternative solution to this problem is to impose further assumptions about the structure of demand as in Klette and Griliches (1996) or De Loecker (2011). The production function can then be recovered from an extended productivity regression. Because we do not observe revenue and industry structure and because standard assumptions about demand made for manufacturing goods are questionable in our context, this type of approach is not the most appropriate here.

in demand rather than by systematic differences in the ease of construction, especially because we condition out many geographical characteristics that may be correlated with supply factors.[7]

We obtain three main results. First, we find that the elasticity of housing production with respect to non-land inputs is roughly constant at 80%. As a first-order approximation, housing is produced under constant returns to scale and is Cobb-Douglas in land and non-land inputs. This said, we can nonetheless formally reject that the housing production function is Cobb-Douglas and constant returns. We can also reject more general functional forms such as the CES. We find evidence of a slight complementarity between land and capital and of small decreasing returns.

In the recent literature, we note the work of Yoshida (2016). He develops a new approach to account for capital depreciation in housing and shows that standard estimates of the elasticity of substitution between land and other inputs can be sensitive to how depreciation is accounted for. Albouy and Ehrlich (2012) estimate a cost function for the production of housing at the city level. Their objective is to explore the determinants and implications of differences in housing productivity across cities. While our focus is to obtain a better measure of the amount of housing, Albouy and Ehrlich (2012) measure it simply using standard hedonics in an intermediate step.

Our work is most closely related to Epple, Gordon, and Sieg (2010) and subsequent work by Ahlfeldt and McMillen (2013) who also treat housing as a latent variable.[8] Like us, they develop a non-parametric estimation of the housing production function using restrictions from theory. We nonetheless differ from their approach in several key respects. First, unlike us, they assume constant returns to scale. For each unit of land, this assumption allows them to express the first-order condition for profit maximisation with respect to capital in terms of the unit price of housing. The latter is not observed but they show that it can be constructed as a monotonic function of the value of housing per unit of land. Our approach shows that imposing constant returns to scale is unwarranted. Second, they rely on different observables, namely housing values per unit of land and land rent per unit instead of land rent and capital for each quantile of parcel size. Third, we implement our approach on very different data: newly constructed houses for an entire country

[7]By using the label 'instrumental variable', we mean the following. We face a simultaneity problem where observed or unobserved confounding factors determine our variables of interest and may bias our results. We attempt to solve this problem by using the variation of appropriate surrogate variables instead of directly using the possibly contaminated variation of our variables of interest. This is consistent with the spirit of extant instrumental variable approaches, even though we do not face the narrow issue of an endogenous explanatory variable in a regression.

[8]In a different vein, Murphy (2015) structurally estimates a dynamic model of housing supply. He seeks to explain how, where, and when housing is produced. His approach relies on a parametric first-order condition for profit maximisation which allows him to recover the marginal cost of construction after having estimated its marginal benefit. The housing supply literature is surveyed in Gyourko (2009).

instead of assessed land values for all houses for a single city, Pittsburgh. Finally, we take steps towards disentangling demand and supply factors in land prices, an issue ignored by Epple *et al.* (2010).[9]

2. Housing: treating output as a latent variable

House builders competitively produce housing services using land T and non-land inputs K, which we refer to as capital for convenience.[10] House builders face a production technology $H(K,T)$ strictly increasing and concave in K. For now, land is exogenously partitioned into parcels of area T where T is distributed over $[\underline{T},\overline{T}]$.

At a given location x, each unit of housing fetches a price $P(x)$. This price reflects the willingness to pay of residents to live at this location. In turn, the demand for locations is assumed to be driven by factors such as employment and shopping accessibility or local amenities as in standard urban models.[11] For a parcel of size T located at x, the builder's profit is $\pi = P(x)H(K,T) - rK - R$, where r is the common user cost of capital and R is the endogenously determined (rental) price of the parcel. Builders are competitive, take the price of housing and of parcels of size T at each location as given, and are left to choose K.

The first-order condition for profit maximisation with respect to capital is,

$$P(x)\frac{\partial H(K,T)}{\partial K} = r\,. \tag{1}$$

The optimal amount of capital inputs that satisfies this condition is given implicitly by $K^* = K^*(P(x),T)$. Because the production function for housing $H(.,.)$ is concave in K, K^* is unique given T. Applying the implicit function theorem to the profit maximisation programme, the concavity of

[9] These differences notwithstanding, our results are broadly consistent with theirs and supportive of unitary elasticity of substitution between land and non-land inputs. When we implement their approach on our data, we find an elasticity of housing with respect to non-land inputs of 0.83.

[10] Non-land inputs are essentially labour and materials, which both get frozen into housing through the construction process. This is consistent with the usual definition of capital.

[11] Monocentric urban models in the tradition of Alonso (1964), Muth (1969), and Mills (1967) define x as the distance to the central business district (CBD) to which residents commute to work at a cost that increases with distance. Both housing services and a composite good enter utility. At the spatial equilibrium, residents at each location choose how much housing and composite good to consume given the local price of housing. Prior to this, they chose their location optimally to maximise utility. At the spatial equilibrium, utility is equalised across locations. The model is often solved using the 'bid-rent approach' by deriving the maximum price that residents are willing to pay subject to achieving a given level of (equilibrium) utility (Fujita, 1989, Duranton and Puga, 2015). Then, given the price of housing at a location, competitive builders choose how much housing to build at this location. Our approach is fully consistent with this standard modelling of land use and urban development but it is more general because we do not need to impose any specific geography. We only use the geography of cities as part of our identification strategy below.

$H(.,.)$ also implies that $\partial K^*/\partial P > 0$. Hence, there is a bijection between the price of housing and the profit-maximising level of capital for any T and we can write $P(x) = P(K^*,T)$.

Free entry implies that the profits from building are dissipated into the price of land so that,

$$R = P(K^*,T)H(K^*,T) - rK^* \equiv R(K^*,T)\,. \tag{2}$$

Note that the price of land in equilibrium is uniquely defined for any K^* and T.

We can insert equation (1) into (2) to eliminate the unit price of housing, which is not observed in the data, to obtain the following partial differential equation:

$$\frac{\partial H(K^*,T)}{\partial K^*} = \frac{r\,H(K^*,T)}{rK^* + R(K^*,T)}\,. \tag{3}$$

For consistency with our empirical work below, this expression may be more intuitively rewritten by transforming its left-hand side into an elasticity:

$$\frac{\partial \log H(K^*,T)}{\partial \log K^*} = \frac{rK^*}{rK^* + R(K^*,T)}\,, \tag{4}$$

where log denotes a natural logarithm. In words, the elasticity of housing with respect to (profit-maximising) capital is equal to the share of capital in the cost of building a house.

Consider that for a given parcel of size T, the desirability of locations varies so that the price of housing is distributed over the interval $\left[\underline{P},\overline{P}\right]$. The optimal level of capital in housing K^* then covers the interval $\left[\underline{K},\overline{K}\right]$ where $\underline{K} = K^*(\underline{P},T)$ and $\overline{K} = K^*(\overline{P},T)$. The solution to the differential equation (4) for a given value of the optimal amount of capital inputs K^* in this interval is obtained by integration and can be written as:

$$\log H(K^*,T) = \int_{\underline{K}}^{K^*} \frac{rK}{rK + R(K,T)} \mathrm{d}\log K + \log Z\,(T)\;. \tag{5}$$

where $Z(T)$ is a positive function equal to $H(\underline{K},T)$. Equation (5) enables the computation of the number of units of housing on parcels of size T knowing the prices of those parcels and the amounts of capital invested to build on these parcels.

The intuition behind this result is relatively straightforward. Locations differ in desirability and thus in their unit price for housing. This price is not observed but it appears in both the optimal capital investment rule described by the first-order condition (1) and in the zero-profit condition (2). We can use the latter equation to substitute for the price of housing in the former and derive differential equation (3), or its log equivalent (4). We then readily obtain log H by integration over log K in equation (5).

To illustrate the workings of equation (5) and check the consistency of our approach, consider first a Cobb-Douglas production function. In this case, the price of land R and the cost of capital used to build a house $r\,K$ are proportional. This implies that the term within the integral is constant.[12] As a result, $\log H$ is proportional to $\log K$. That is, we retrieve a Cobb-Douglas form.

To take another example, assume now that the production function enjoys a constant elasticity of substitution between land and capital equal to two. In this case, profit maximisation implies that capital inputs should increase with the square of the price of parcel of size T. Integrating the share of capital as in equation (5) implies that the production of housing is proportional to $(\sqrt{K}+b)^2$ where b is a constant. This functional form is indeed the generic functional form for a CES production function with an elasticity of substitution equal to two when a factor (land) is held constant.

An important assumption of our model is that the price of land for a parcel is affected by its location x only through the price that residents are willing to pay to live at this location. That is, the price of land is determined entirely by the demand side. Put differently, our approach so far does not allow for a parcel characteristic y to affect the production technology directly. To understand the implications of supply differences across parcels, let us consider first a simple example where all parcels are of unit size, the demand for housing is the same at all locations, $P(x) = P = 1$, and the price of capital inputs is normalised to unity, $r = 1$. Assume that housing is produced according to $H(K,y) = \frac{1}{a}K^a\,y^{1-a}$ where the unobserved characteristic y measures the ease of construction. Then, in equilibrium, capital is given by $K(y) = y$ and parcel prices capitalise the ease of construction, $R(y) = \frac{1-a}{a}y = \frac{1-a}{a}K$. Using equation (5) to estimate the value of housing, we would obtain that the production of housing is proportional to K instead of being proportional to K^a.[13]

More generally, assume that parcels are now characterised by two location characteristics, x and y. The characteristic x still affects the price that residents are willing to pay, $P(x)$, while y affects the production of housing directly, which is now given by $H(K,T,y)$. The analogue to the first-order condition (1) is $P(x)\partial H(K,T,y)/\partial K = r$. The zero-profit condition also implies that y affects the price of land directly: $R(K^*,T,y) = P(x)H(K^*,T,y) - rK^*$. The partial differential

[12] This property was already noted by Klein (1953) and Solow (1957).

[13] Note that this problem of missing variable is worse than in standard cases because it creates a bias even when the missing characteristic y is uncorrelated with P as illustrated by our example.

equation analogous to (4) is:

$$\frac{\partial \log H(K^*,T,y)}{\partial \log K^*} = \frac{rK^*}{rK^* + R(K^*,T,y)} . \tag{6}$$

It can be solved only for a given y. Integrating as we do in equation (5) ignoring y will be problematic since y will be correlated with both the quantity of housing H and the price of land R. Locations with a particularly good y will both be able to generate more housing for a given amount of capital and face a higher price for land. Below, we develop an instrumental-variable approach to circumvent this problem.

Before going forward, some of our other assumptions must be discussed further. First, we assume non-increasing marginal returns to capital. This is arguably an appropriate assumption for newly constructed single-family homes. Second, because of the ease of entry in this industry and the absence of fine product differentiation, our assumption of competitive builders also strikes us as reasonable.[14] Third, at every location, the unit price of housing is taken as given by competitive builders. We thus implicitly assume an integrated housing market and (fairly) homogenous preferences. This is defensible in our empirical application below since we ignore outliers and in robustness checks, we consider the construction cost of houses at early stages of completion and conduct separate estimations for different socio-economic groups of buyers. Finally, parcels are exogenously determined. Treating land as a fixed input is reasonable in France where zoning rules usually prevent the subdivision of existing parcels in residential areas. We discuss further identification issues below with our empirical strategy.

3. Estimation of the housing production function

There are four main steps to our empirical approach. We first kernel-smooth the price of parcels R for pairs of capital K and parcel size T. Next, we estimate non-parametrically the amount of housing $H(K)$ for a given T using equation (5). We then describe the shape of $H(K)$ by means of simple regressions. The same approach is implemented with and without conditioning out supply factors that may affect R, K, and T prior to smoothing.

[14] A search on the French yellow pages (`http://www.pagesjaunes.fr/`) yields 1783 single-family house builders for Paris (largest urban area with population above 12 million), 111 for Rennes (10th largest urban area with population 654,000), and still 38 for Troyes (50th largest urban area with population 188,000). (Search conducted on 21st May 2013 looking for 'constructeurs de maisons individuelles' – builders of single family homes – typing 'Ile-de-France' to capture the urban area of Paris, 'Rennes et son agglomération', and 'Troyes et son agglomération' for the other two cities.)

3.1 Implementation

Equation (5) computes housing by integrating a cost share over K for a given T. In the data, the price of parcels of a given size T is observed only for some values of capital, not for the entire continuum. For any pair (K,T) for which no parcel price is observed, we can rely on observations with slightly larger and slightly lower values of K and/or T. That is, we can estimate the price of land for every value of K and T using a kernel non-parametric regression. The kernel we use is the product of two independent normals and the bandwidth is computed using a standard rule of thumb for the bivariate case (see Silverman, 1986).[15] The estimated price of land is given by the following formula:

$$\widehat{R}\left(K,T\right) = \sum_i \omega_i R\left(K_i,T_i\right) \text{ with } \omega_i = \frac{L_{h_K}\left(K-K_i\right) L_{h_T}\left(T-T_i\right)}{\sum_i L_{h_K}\left(K-K_i\right) L_{h_T}\left(T-T_i\right)}, \tag{7}$$

where N is the number of observations, $L_h\left(x\right) = \frac{1}{h} f\left(\frac{x}{h}\right)$ with $f\left(\cdot\right)$ the density of the normal distribution, and $h_X = N^{-1/6}\sigma\left(X\right)$ with $\sigma\left(X\right)$ the empirical standard deviation for variable X computed from the data. This kernel estimator has the property of making $R(K,T)$ unique, which is requested by our model.

Note also that equation (5) involves the computation of an integral for which the lower bound is the lowest value of the profit-maximising capital. In practice, we can potentially use any value of capital as lower bound, $\underline{K}$, but there is a trade-off. A small value for the lower bound will allow us to study the variations of the housing production function over a wide range of values for capital inputs but this may come at the cost of being in a region where there are few observations. In our work below, we restrict attention to observations above the first decile (and below the ninth decile) to estimate the production of housing.

After smoothing the data and estimating the production of housing using equation (5), we regress the non-parametrically estimated housing production on capital inputs. We estimate these regressions to describe how housing production varies with capital. For instance, under Cobb-Douglas, for any fixed T, there should be a linear relationship between the log of the amount of housing produced and log capital. In section 6, we further assess a variety of alternative functional forms by comparing our non-parametric estimates of housing production to estimates for which

[15] Alternatively, we could consider that the integrand $\frac{rK}{rK+R(K,T)}$ is computable only at the observed values of capital and recover that integrand for other value of capital using a kernel non-parametric regression of the integrand.

we impose a functional form in the first place (instead of kernel-smoothing the data).[16]

3.2 *Dealing with supply-side unobserved heterogeneity*

As already mentioned, our approach relies on the fact that the price of a parcel should only reflect the price that housing can fetch on this parcel. However, the price of a parcel may also reflect the ease of construction. For instance, a parcel may be more costly to develop because of a steep slope or because it is harder to excavate. For a given price of housing at this location, this parcel will be worth less in equilibrium. More generally, consider a location characteristic y that affects the optimal investment in capital and thus the price of a parcel. By equation (6), we can only appropriately estimate the production function for housing for a given y (which may not be observed).

To deal with that problem, our empirical strategy is to purge our variables of interest, R and K, from the effects of supply characteristics by relying only on the variation in the demand for housing across locations. In practice consider the following regression:

$$\log R_i = X_i\, a^R + Y_i\, b^R + f^R(T) + \epsilon_i^R\,, \tag{8}$$

where X is a vector of location characteristics that (are assumed to) affect the demand for housing, Y is a vector of location characteristics that (are assumed to) affect the supply of housing, and $f^R(T)$ is a potentially non-parametric function of T. The vector X is the empirical counterpart of the location effect x in our framework above while Y is the empirical counterpart to y. To estimate $f^R(T)$, we use indicator variables for every size centile. Then, we can compute a predicted land price $\widehat{R}(K,T)$ which depends only on demand characteristics and not on supply characteristics: $\widehat{R}(K,T) = \exp(X\,\hat{a}^R + \overline{Y}\,\hat{b}^R + \hat{f}^R(T) + (\hat{\sigma}^R)^2/2)$ where $\overline{Y}\,\hat{b}^R$ is the mean effect of supply characteristics and $\hat{\sigma}^R$ is the estimator of the standard deviation of the error term of the regression described by equation (8). We can then use predicted instead of actual parcel prices in the empirical counterpart to equation (5).

The location characteristics Y that affect the price of parcels through the supply of housing will also affect the optimal use of capital and, in turn, bias our estimates. Hence, we also want to estimate the following regression analogous to equation (8) for capital:

$$\log K_i = X_i\, a^K + Y_i\, b^K + f^K(T) + \epsilon_i^K\,. \tag{9}$$

[16]We prefer this approach to more standard specification tests that tradeoff a measure of goodness of fit against the number of explanatory variables using arbitrary weights.

Like with parcel prices, we can then compute a predicted value for capital $\widehat{K}$ which depends only on demand characteristics: $\widehat{K} = \exp(X\,\hat{a}^K + \overline{Y}\,\hat{b}^K + \hat{f}^K(T) + (\hat{\sigma}^K)^2/2)$ where $\overline{Y}\,\hat{b}^K$ is the mean effect of supply characteristics and $\hat{\sigma}^K$ is the estimator of the standard deviation of the error term in equation (9). We can then use predicted instead of actual capital in the empirical counterpart of equation (5).

As sources of exogenous demand variation, we use the urban area to which a parcel belongs and the distance to its centre. This is consistent with monocentric urban models in the tradition of Alonso (1964), Muth (1969), and Mills (1967) where the price of housing, land prices, and capital investment at each location are fully explained by the distance to the centre and city population. We also use local measures of income which also play a role in more elaborate models of urban structure with heterogeneous residents (Duranton and Puga, 2015).

This said, we worry that the urban area of a parcel or its distance to the centre may be correlated with the ease of building. For instance, construction labour may be cheaper in some cities (Gyourko and Saiz, 2006) or terrains characteristics may vary systematically with distance to the centre. This is why we also include a number of geographic municipal characteristics as part of our vector of supply characteristics Y to be conditioned out. In addition, we can condition out the local wage of blue-collar workers in the construction industry from urban area fixed effects.[17]

More specifically, to estimate equations (8) and (9), we include urban area fixed effects, distance to the centre (allowing the effect to vary across urban areas), three municipal socioeconomic characteristics (log mean income, its standard deviation, and the share of population with university degree), and seven geological variables (ruggedness, and three classes of soil erodability, soil hydrogeological class, and soil dominant parent material). In our demand vector, we include urban area fixed effects (after conditioning out local wages for construction workers), distance to the centre, and municipal socioeconomic characteristics. We verify that our results are robust to using more restrictive or more inclusive definitions of the demand determinants.[18]

Finally, recall that our model takes parcel size T as given. The location characteristics that affect the cost of construction may also affect parcel size. For instance, parcels may be larger where construction is more costly. This suggests applying the same approach as in equation (8) to parcel

[17]Wages in the construction industry in an urban area cannot be directly included in the regression because they would be collinear with urban area fixed effects but we can regress urban area fixed effects on wages in the construction industry and retain the estimated residual.

[18]We retain the same rich specification throughout but vary the composition of X. We also experimented with specifications with fewer control variables and obtained similar results. We do not report these results here.

size and estimate:

$$\log T_i = X_i\, a^T + Y_i\, b^T + \epsilon_i^T\,. \tag{10}$$

The resulting predicted value $\widehat{T} = \exp(X\,\hat{a}^T + \overline{Y}\,\hat{b}^T + (\hat{\sigma}^T)^2/2))$ can then be used in our estimation of equation (5) instead of the actual parcel size. When we also allow for T to be endogenous, we need to amend equations (8) and (9) above and consider instead the following two equations

$$\log R_i = X_i\, a^R + Y_i\, b^R + \epsilon_i^R\,, \tag{11}$$

$$\log K_i = X_i\, a^K + Y_i\, b^K + \epsilon_i^K\,. \tag{12}$$

where we no longer include T as determinant of R and K.

Our identification strategy relies on the same principle as standard instrumental variables approaches because it uses the (conditional) variation of surrogate variables, such as the urban area of a parcel in our case, rather than the entire variation of the variable of interest. While the principle is the same, our implementation differs considerably from standard two-stage least-squares procedures. Our objective is to provide a non-parametric estimate of housing, H, as a function of capital, K, for a given parcel size, T. Given that observed and unobserved supply characteristics of parcels are expected to affect capital, land values, and parcel size, our 'first stage' generates predicted values for three variables, capital $\hat{K}$, land values $\hat{R}$, and parcel size $\hat{T}$. We then estimate housing production as when we use gross values of K, R and T.

Although we may use as little as one instrument for three variables in the first step, the effect of capital on housing production is nonetheless identified with our procedure. We are not in a situation where we attempt to estimate the effect of three endogenous variables with one instrumental variable. Instead, we use surrogate variables to instrument a triplet (K,R,T) (or only the pair (K,R)) and not three different explanatory variables. In effect, we use predicted values of K and R to estimate $\log H$ (given actual or predicted T).

To address more directly the issue of differences in labour costs and other possible forms of supply heterogeneity across cities, we would like to estimate a separate housing production function for each urban area separately. Except for the largest French cities, the number of parcel transactions in a typical urban area is unfortunately too small to do this. We nonetheless divide urban areas into size classes and perform a separate estimation for each size class.

Measurement error on capital inputs K or land prices R may also affect our estimation. Measurement error is dealt with in two different ways in our approach. First, as mentioned above, we kernel-smooth parcel price data. Second, our IV approach will use predicted land prices which are less prone to measurement error than observed land prices.

As already highlighted, our model assumes an integrated market for housing. This is what allows us to think in terms of units of housing that can be measured and compared across houses. A full relaxation of the integrated market assumption would involve considering each house as uniquely differentiated variety over which residents have idiosyncratic preferences. When all residents value all houses differently, the notion of a common unit of housing is no longer well defined. While our approach is unable to deal with such extreme cases, in a robustness check, we consider housing markets segmented across socio-economic groups using information about the characteristics of the buyers.

To address product differentiation in housing, we can also use the fact that the reported construction costs are for one of three levels of completion ('fully finished', 'ready to decorate', and 'structure completed'). Should idiosyncratic preferences affect building costs, we expect they will matter more for houses at a more advanced state of completion. We can thus assess the importance of demand heterogeneity indirectly by comparing results across the different stages of completion.

We need to keep in mind that housing construction is tightly regulated in France as in many other countries. The three main regulatory instruments are (i) the zoning designation, (ii) the maximum intensity of development, and (iii) severe restrictions on parcel division.[19] The zoning designation indicates whether a parcel can be developed and, if yes, whether this can be for residential purpose. Given that we only observe parcels with a development permit for a single-family home, this creates no further issue beyond the fact that we estimate the production function for single-family homes in parcels designated for that purpose. Turning to the maximum intensity of development that applies to a parcel, this information is not centrally collected by the French government. Although we emphasise that the quantity of housing is not solely determined by the square-footage of a house, we nonetheless acknowledge that we estimate the production function for housing under prevailing restrictions on residential development. Absent regulations, single family homes would perhaps be different from what they are under the current regulatory

[19]The maximum intensity of development is essentially a maximum floor-to-area ratio (FAR) regulation. In France, it is referred to as the 'coefficient d'occupation des sols' (COS). This regulation is subject to national guidelines but can be adjusted locally by the municipality. Other regulations such as minimum parcel size or an obligation to follow a local style with more or less stringency also often apply.

regime.[20] However, we are interested in estimating how land and capital inputs are transformed into housing under the current regulatory constraints. While knowing how regulations affect the production of housing is certainly a question of interest, this is not one that we can answer here.

Finally, we acknowledge further limitations of our framework. First, as already noted, the price of housing on a parcel is only determined by its location, not by the intensity of development on this parcel.[21] The second issue is that single-family homes are indivisible (by definition) and the price per unit of housing may decline with the quantity of housing offered by a house, at least beyond a certain quantity. Second, our model is static and ignores that housing development is, to a large extent, an irreversible decision. In turn, this implies that the price of vacant land may include the option value to develop it.[22] Note also that we estimate the production function for one house not for builders who may build several houses. In particular, there might be sizeable economies of scale arising from being able to build many houses at the same location at the same time.

3.3 *Implementation*

To implement our approach, for each of the nine deciles of parcel size, we consider 900 values of capital uniformly distributed over the interval defined by the first and last deciles of capital within the parcel size decile at hand. This generates a fine grid of 8,100 (K,T) points. We first estimate parcel prices at any point of the grid using equation (7) with up to 386,181 observations for the entire country in our data set. Then, for each of the nine deciles of parcel size, the housing production function is estimated using equation (5) by summing over the values of capital within the parcel size decile at hand, using trapezoids to approximate the integral. By construction, for any parcel size decile, the lower bound of integration $\underline{K}$ corresponds to the bottom decile and the maximum upper bound $\overline{K}$ to the top decile of capital values. This avoids having our estimations influenced by potential outliers or by observations that belong to a different market segment such as luxury homes. We obtain 8,100 values of housing production, corresponding to 900 values of capital for each of the nine parcel size deciles.

[20] To be concrete, consider that two technologies with very different production functions are available to build housing. If one technology is banned through regulations, we can only learn about the second.

[21] Demand for housing on a parcel might decline with the intensity of development on this parcel. This is certainly an issue for multi-family buildings. It should be less problematic with single-family homes.

[22] See Capozza and Helsley (1990) and subsequent literature or Duranton and Puga (2015) for a review.

Before turning to our results, several implementation issues must be discussed. Our model relies on the rental value of land and the rental value of capital inputs. The data we use only report the price of land and the cost of construction. Using stocks (transaction values) instead of flows (rental values) makes no difference to our approach when the user cost of land is the same as the user cost of capital. This is not the case when the user costs differ across factors.[23] Following Combes *et al.* (2016), we take the annual user cost of capital to be 6%. This reflects a long term interest rate of 5% and a 1% annual depreciation.[24] For the user cost of land, we take a value of 3% per year.[25] To make parcel prices and capital investments comparable over time, we also correct for year effects which we obtain from regressions of log parcel prices and log capital on year fixed effects.

Finally, confidence intervals are estimated by bootstrap. At each iteration, a random sample is drawn with replacement from the universe of all transacted land parcels. Parcel prices are recomputed at each point of our (K,T) grid using kernel non-parametric regressions before re-estimating housing production. The distribution of values for housing production at each point of the grid is then recovered and confidence intervals can be deduced.

4. Data

The observations in our data are transacted land parcels with a building permit that are extracted from the French Survey of Developable Land Price (*Enquête prix des terrains à bâtir*, *EPTB*). This survey is conducted every year in France since 2006 by the French Ministry of Ecology, Sustainable Development, and Energy. The sampling frame is drawn from Sitadel, the official land registry, which covers the universe of all building permits for detached houses. The survey selects building permits for owner-occupied, single-family homes. Permits for extensions to existing houses are excluded. A small fraction of parcels (less than 3% in 2006) also has a demolition permit. Our

[23] This is very much in the spirit of the user-cost correction first proposed by Poterba (1984).

[24] In the French national accounts, housing depreciation can be computed as the difference between investment in housing and the increase in housing stocks. According to Commissariat Général au Développement Durable (2012), this difference in 2009 was about 15 bn Euros, which corresponds to slightly less than 1% of GDP or just below 0.6% of the value of the stock. This is arguably a lower bound as much housing maintenance falls under home production and is not accounted for in national accounts.

[25] As estimated by Combes *et al.* (2016), the elasticity of land prices with respect to local income is slightly above one while the elasticity of the price of land with respect to population is slightly below one. A 1% annual increase in income and a 1% annual increase in population (the mean urban population growth in France in the recent past) thus imply an about 2% annual appreciation of land prices to be deducted from the long run interest rate of 5%.

study period is from 2006 to 2012.[26] Originally, about two thirds of the transactions with permits were sampled. The survey became exhaustive in 2010. This survey is mandatory and the response rate, after one follow-up, is above 75%. Annually, the number of observations ranges from 48,991 in 2009 to 127,479 in 2012.

While it is possible to get new houses built in many ways in France, the arrangement we study covers a large fraction of new constructions for single-family homes.[27] Households typically first buy constructible land, obtain a building permit, and get a house built by themselves, through a general contractor, or an architect. Only about 20% of new houses are 'self-built' as French law requires the use of a general contractor or an architect for constructions above 100,000 Euros. Getting a new house by first first buying land subsequently signing a contract with a builder is fiscally advantageous as it avoids paying stamp duties on the structure.[28] This arrangement also greatly reduces financing constraints for house builders and lowers their risks.

For each transaction, we know the price of the parcel, its size, whether it is 'serviced' (i.e., has access to water, sewerage, and electricity), its municipality, how it was acquired (purchase, donation, inheritance, other), some information about its buyer, whether the parcel was acquired through an intermediary (a broker, a builder, another type of intermediary, or none), and some information about the house built, including its cost (but with no breakdown between material and labour). The notion of building costs may be ambiguous but we know whether the reported cost reflects the cost of a fully decorated house, the cost of a house prior to decoration (i.e., excluding interior paints, light fixtures, faucets, kitchen cabinets, etc), or only the cost of the structure. Ready-to-decorate houses represent the large majority of our observations (72%). We only retain parcels that were purchased and ignore inheritances and donations. We also appended a range of municipal and urban area characteristics described in Appendix A.

Table 1 provides descriptive statistics for all our main variables. The first interesting fact is the considerable variation in parcel size, total construction costs, and parcel price per square meter. A parcel at the top decile is about four times as large as a parcel at the bottom decile. Interestingly, for

[26] It is important to keep in mind that, unlike in the US, there was no housing burst in France during this period and that the heterogeneity of housing price fluctuations across cities was far from being as extreme as in the US.

[27] The consultancy Développement-Construction reports between 120,000 and 160,000 new single-family homes per year during the period (`http://www.developpement-construction.com/`). These magnitudes closely coincide with the number of observations in later years after accounting for the response rate.

[28] This tax avoidance is only partially offset by a VAT abetment for construction costs. Stamp duties in France currently represent about 5.8% of the value of the transaction exclusive of notary and various ancillary fees.

Table 1: Descriptive Statistics

Variable	Mean	St. deviation	1st decile	Median	9th decile
Entire country:					
Parcel size	1,156	947	477	883	2,079
Construction cost	127,551	55,003	78,440	115,000	190,667
Parcel price	63,387	58,164	19,673	50,000	120,000
Parcel price per m^2	80	86	14	58	166
Urban areas:					
Parcel size	1,048	821	449	820	1,883
Construction cost	131,616	57,599	80,140	118,000	199,750
Parcel price	73,115	62,518	27,017	58,271	135,000
Parcel price per m^2	96	94	22	72	192
Greater Paris:					
Parcel size	839	673	329	665	1,493
Construction cost	151,298	73,727	89,173	132,850	236,605
Parcel price	142,010	108,598	69,155	124,419	220,000
Parcel price per m^2	237	193	67	182	466

Notes: The sample contains 386,181 observations for the entire country and 218,657 observations for urban areas. Parcel sizes are in square meters. Parcel prices and construction costs are expressed in 2012 Euros, using the French consumer price index.

construction costs, the corresponding inter-decile ratio is only about 2.4 whereas for parcel prices per square meter, it is nearly 12. The second interesting feature of the data highlighted in table 1 is that this variation does not only reflects a rural / urban gap. Even when we consider only transactions from Greater Paris, we still observe considerable variation in parcel price per square meter.

A possible worry here is that the construction costs reported by surveyed households may not accurately reflect how much they actually paid to get their house constructed and much of the variation reported in table 1 may just be measurement error. We first note that contracts with general contractors usually include a small number of installments and we expect households to remember the headline figure. We can investigate this issue further using data from the Survey of Costs of New Dwellings (Enquête sur le Prix de Revient des Logements Neufs). This is a detailed survey of builders that forms the basis of the French construction price index, which, in turn, is used to index rent increases for residential rented properties, rents for parking spots, and until quite recently commercial leases. From the second quarter of 2010 through the fourth quarter of 2012, we could match all 2,336 observations in this survey that should also have been included in our main data using the building permit identifier. Reassuringly, the correlation between the two

Figure 1: Probability distribution function of the relative distance for new constructions, French urban areas 2006-2012

Notes: All years of data used. 218,657 observations. For each new construction, we compute the distance between the centroid of its municipality and the centroid of its urban area and divide by the greatest observed distance for any new construction in this urban area.

measures of housing costs is 0.83 both in levels and in logs.

Rather than reflecting mis-measurement, the variation in prices within cities is to a large extent driven by the fact that new constructions in French urban areas are, in their large majority, in-fills that occur everywhere in their urban area, from more expensive central locations to cheaper peripheral ones. To illustrate this, figure 1 represents the probability distribution function of the relative distance to the center of their urban area for new constructions. Less than 2% of the observations are beyond 95% of the maximum distance to the centre and the modal distance for these constructions is at about 40% of the maximum distance to the centre of the urban area. Consistent with the preponderance of in-fills, another data source, the Survey of Commercialisation of New Dwellings (Enquête sur la Commercialisation des Logements Neufs) indicates that about 10% or less of building permits for single-family homes are for groups of five or more.[29]

To compute non-parametric estimates of the production of housing, we use predicted parcel prices estimated with equation (7) using a kernel non-parametric regression. To measure how well these predicted parcel prices capture actual prices, we compute the following measure of goodness

[29]This additional exhaustive survey mentions 7,915 single-family homes in developments of five or more put on the market over a 12 months period from mid-2014 to mid-2015 (`http://www.developpement-durable.gouv.fr/IMG/pdf/CS667-2.pdf`). We have 95,381 single-family homes in the EPTB sampling frame for 2014. While these two sources are not directly comparable given the slight differences in timing, they are nonetheless supportive of fact that most new single-family homes are built as part of small-scale developments.

of fit: $\mathcal{R}^2 = 1 - \frac{\sum_i (\hat{R}_i - R_i)^2}{\sum_i (R_i - \overline{R})^2}$ where $\overline{R}$ is the mean parcel price and $\hat{R}_i$ is the parcel price predicted non parametrically at the observed values of capital and parcel size (K_i, T_i) using the same kernel as in equation (7). We also compute the correlation between actual and predicted parcel prices. For the rule-of-thumb bandwidth that we use in most of our estimation the (pseudo) $\mathcal{R}^2$ is 0.20 and the correlation between actual and predicted parcel prices is 0.45. Using instead bandwidths that are half, a quarter, and a tenth of the rule-of-thumb bandwidth leads to $\mathcal{R}^2$ of 0.25, 0.32, and 0.43, respectively. For these alternative bandwidths, the correlations between actual and predicted parcel prices are 0.50, 0.57, and 0.66, respectively.[30] We verify below that our choice of bandwidth does not affect our results.

5. Results

5.1 Main results using the raw data

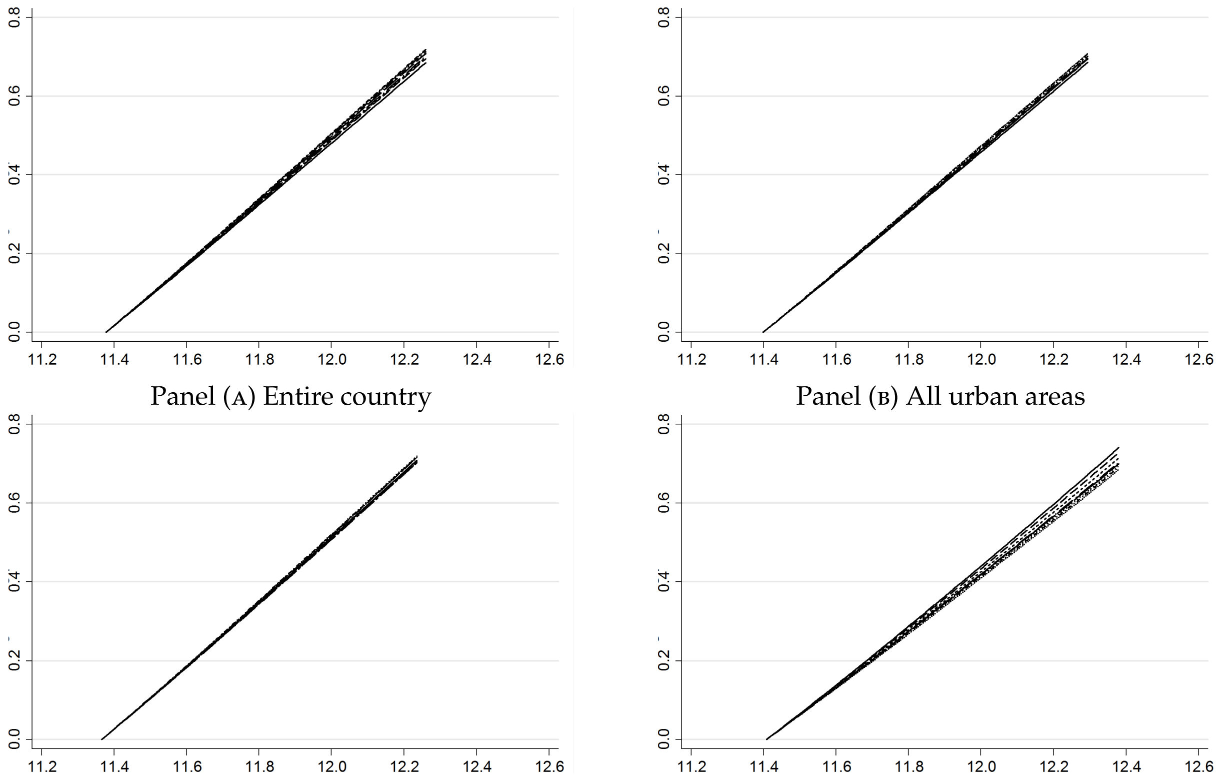
Before looking at formal estimation results, it is useful to visualise our non-parametric estimations. Each panel of figure 2 plots the estimated log production of housing, $\log H$, as a function of capital investment, $\log K$, for every decile of parcel size, T.[31] This is the empirical counterpart to equation (5). Panel (A) represents the production function for housing for the entire country while panels (B), (C), and (D) do the same for all urban areas, small urban areas with population between 50,000 and 100,000 and large urban areas with population above 500,000 (bar Paris), respectively. We obtain similar patterns for other city size classes, as confirmed by the regression reported below.

Although we must remain cautious when visualising these results, several remarkable features emerge from figure 2. First, as might be expected, housing production always increases with capital. More specifically, log housing is apparently a linear function of log capital with a slope of about 0.80. This is of course consistent with a Cobb-Douglas function with a constant elasticity of housing production with respect to capital of about 0.80. Second, the relationship between $\log H$ and $\log K$ appears very similar across all deciles of parcel size. Although we can identify the production function of housing only for each quantile of parcel size, the minimal differences that appear across deciles in figure 2 indicate a similar elasticity of housing production with respect to

[30] While smaller bandwidths lead to a better fit at the observed values of capital and parcel size, they are potentially problematic for some points of our grid since they may not allow the use of enough observations around these points to obtain accurate predicted parcel prices. This is why we smooth in the first place.

[31] Epple *et al.* (2010) end their analysis with a similar figure.

Figure 2: log housing production as a function of log capital investment, non-parametric estimates



Panel (A) Entire country

Panel (B) All urban areas

Panel (C) Urban areas, 50,000-100,000

Panel (D) Urban areas, more than 500,000 (excl. Paris)

Notes: The log amount of housing production is represented on the vertical access and the log amount of capital investment is represented on the horizontal access. To ease the comparison across deciles of parcel size, we normalise $\log H(\underline{K})$ to zero for all deciles. 386,181 observations for the entire country and 218,657 for urban areas.

capital on small and big parcels alike. The last important feature of figure 2 regards the differences across panels. While the relationship between $\log H$ and $\log K$ is very much the same across the first three panels, the last panel for large cities is modestly different with more dispersion across deciles and a slightly lower slope.

We next turn to regressions to describe these non-parametric estimates more precisely. Our first set of results is reported in panel (A) of table 2 where, for each decile of parcel size, we regress our non-parametric estimates of the log production of housing on log capital for observations located in urban areas. Each regression relies on 900 observations obtained after smoothing parcel prices as per equation (7) at values of capital between its first and last deciles. Each column of table 2 corresponds to a separate decile of parcel size. The estimated capital elasticity of housing for the first decile is 0.77. It is 0.78 for the second to the fifth decile, 0.79 for the seventh and eighth, and finally 0.80 for the last decile. While these elasticities are not exactly constant across

Table 2: log housing production in urban areas, OLS by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A)									
log (K)	0.768^a	0.779^a	0.780^a	0.779^a	0.782^a	0.788^a	0.790^a	0.794^a	0.796^a
	(0.00064)	(0.00053)	(0.00064)	(0.00066)	(0.00081)	(0.0011)	(0.0012)	(0.0016)	(0.0020)
	[0.00071]	[0.00059]	[0.00063]	[0.00073]	[0.00085]	[0.0011]	[0.0012]	[0.0016]	[0.0019]
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Observations	900	900	900	900	900	900	900	900	900
Panel (B)									
log (K)	0.379^a	0.282^a	0.217^a	0.274^a	0.288^a	0.367^a	0.502^a	0.480^a	0.498^a
	(0.028)	(0.022)	(0.026)	(0.031)	(0.040)	(0.052)	(0.070)	(0.084)	(0.087)
	[0.030]	[0.021]	[0.024]	[0.031]	[0.040]	[0.051]	[0.065]	[0.082]	[0.090]
$[\log(K)]^2$	0.016^a	0.021^a	0.024^a	0.021^a	0.021^a	0.018^a	0.012^a	0.013^a	0.013^a
	(0.0012)	(0.00092)	(0.0011)	(0.0013)	(0.0017)	(0.0022)	(0.0030)	(0.0034)	(0.0037)
	[0.00125]	[0.00088]	[0.00100]	[0.0013]	[0.0017]	[0.0021]	[0.0028]	[0.0035]	[0.0038]
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Observations	900	900	900	900	900	900	900	900	900

Notes: OLS regressions with a constant in all columns. Bootstrapped standard errors with 100 bootstraps in parentheses and with 1,000 bootstraps in squared parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%. Non-parametric estimates of housing production rely on 218,657 observations.

deciles, the differences remain small. Interestingly, the capital elasticity of housing is estimated to be larger in larger parcels. This is consistent with the production function of housing being log super-modular. For a constant-elasticity of substitution production function, this implies land and capital being (weakly) complement and an elasticity of substitution between land and capital just below one. Because these estimates are subject to a number of identification worries, we refrain from further conclusions for now but note that the differences in the production function across parcels of different sizes are economically small. Importantly, we also note that our linear regressions provide a near perfect fit as the R^2 is always above 0.999.[32]

Panel (B) of table 2 replicates the regressions of panel (A) adding the square of log capital as explanatory variable. We note that the estimated coefficient of the quadratic term is significant in all regressions with a coefficient between 0.012 and 0.024. Hence, the production function for housing is not strictly log linear in capital but log convex. Because log capital typically varies between about 11.4 at the bottom decile and, 12.2 at the top decile, this log convexity implies that

[32] Recall that we work with smoothed data, which condition out idiosyncratic variation. To be clear, this R^2 does not measure how well our regression fits the raw data but how well the functional form imposed by the regression fits the non-parametric estimate of the housing production function.

Table 3: log housing production, OLS by class of urban area population

City size class	Country	Urban areas	0-50	50-100	100-200	200-500	500+	Paris
Panel (A)								
log (K)	0.805^a	0.784^a	0.832^a	0.822^a	0.814^a	0.785^a	0.730^a	0.700^a
	(0.00060)	(0.00063)	(0.0014)	(0.0012)	(0.0011)	(0.0011)	(0.0014)	(0.0025)
Panel (B)								
log (K)	0.315^a	0.365^a	-0.075^a	0.038^a	0.230^a	0.068^a	-0.091^a	-0.002
	(0.028)	(0.025)	(0.062)	(0.052)	(0.039)	(0.038)	(0.050)	(0.070)
$[\log (K)]^2$	0.021^a	0.018^a	0.038^a	0.033^a	0.025^a	0.030^a	0.034^a	0.029^a
	(0.0018)	(0.0011)	(0.0026)	(0.0022)	(0.0017)	(0.0016)	(0.0021)	(0.0028)

Notes: OLS regressions with decile fixed effects in all columns. 8,100 observations for each regression. The R^2 is 1.00 in all specifications. Bootstrapped standard errors in parentheses. a, b, c: significant at 1%, 5%, 10%.

the capital elasticity of housing is only about 0.02 larger for houses built at the top decile of capital relative to houses built at the bottom decile. While the housing production function is log convex, this log convexity is minimal and the differences in the capital elasticity between the largest and smallest houses are tiny.

All the coefficients reported in table 2 are highly significant. This table reports two series of standard errors with 100 and 1,000 bootstraps, respectively. Because taking 1,000 bootstraps does not make any substantive difference and because these bootstraps are computationally very intensive, we only report standard errors computed from 100 bootstraps in what follows.

Panel (A) of table 3 regresses again the log of estimated housing production on log capital but this time considers different samples of new constructions corresponding to different geographies. In each regression, all the parcel size deciles are lumped together and decile fixed effects are included. The first column considers the entire population of transactions. The estimated capital elasticity of housing is 0.80. Column 2 considers only observations from urban areas and estimates a marginally lower elasticity of 0.78. The following six columns consider urban areas of increasing sizes. For the smallest urban areas with population below 50,000 the estimated capital elasticity is 0.83. This elasticity is 0.73 for large urban areas with population above 500,000 and 0.70 for Paris. Because land is more expensive in larger cities, these results are again consistent with a modest complementarity between land and capital in the production of housing. Panel (B) of table 3 repeats the same exercise as panel (A) adding a quadratic term for log capital. Just like panel (B) of table 2 it provides evidence of a modest log convexity.

Before turning to our instrumented results, we assess the robustness of the results we have obtained so far through four different checks. First, recall that houses are built for specific buyers who may have idiosyncratic preferences that affect construction costs. Because the information about construction costs is for one of three levels of completion, we can compare results across these levels of completion (fully finished units, ready-to-decorate units, and units with only a completed structure). Any unobserved heterogeneity associated with the customisation of houses should have a greater effect on fully finished units than on bare-bone structures. Table 11 in Appendix B duplicates panel (A) of table 2 but splits observations by level of completion. Unsurprisingly, we find a slightly higher capital elasticity for more finished houses that reflects their greater capital intensity. For the median parcel, the capital elasticity is 0.793 for fully finished units, 0.779 for ready-to-decorate units, and 0.755 for units with only a completed structure. The corresponding elasticity in table 2 is 0.782. Importantly, for all levels of completion, we find again a modestly increasing capital elasticity as we consider higher parcel size deciles as in table 2.

The segmentation of housing markets may imply another form of unobserved heterogeneity. While we cannot track the heterogeneity of houses directly, it may be reflected in the heterogeneity of buyers. We can use information regarding the buyer's occupation and split the sample of transactions by buyers' occupation: executives, intermediate occupations, and clerical and blue-collar workers. We report results for these three groups in table 12 in Appendix B. The differences between occupational categories are small. For the median parcel, the capital elasticity is 0.771 for executives, 0.781 for intermediate occupations, and 0.791 for clerical and blue-collar workers with the same general pattern of modestly increasing elasticities as we consider larger parcels.

As noted above, the details of our smoothing procedure affects the quality of our predictions for land prices. To verify that our results are not affected by our choice of bandwidth, table 13 in Appendix B repeats the estimations of table 2 for bandwidths equal to a half, a quarter, and a tenth of the rule-of-thumb bandwidth, respectively. When regressing log housing production on log capital, the results are virtually identical for all bandwidths. When we also include the square of log capital as explanatory variable, the results are again the same except, perhaps, for the smallest bandwidth at the top decile of parcel sizes.[33] Reassuringly, these results show that we

[33] For larger parcels, there is more variation in capital so that observations are sparser in the upper decile. Recall that taking a bandwidth that is only 10% of what is suggested by the rule-of-thumb is potentially problematic as 'holes' in the data are no longer properly smoothed away.

need to consider extreme forms of under-smoothing before running into these problems. The main conclusion is that our results are not affected by our choice of bandwidth.

Finally, table 14 in Appendix B duplicates again table 2 but does not apply our user cost correction of 6% for structure and 3% for land. Because we no longer account for the depreciation of capital and the appreciation of land, we estimate a lower coefficient on capital to 0.642 for the median parcel. This elasticity should now be interpreted as the elasticity of the housing stock instead of the elasticity of housing services. The lack of user cost correction does not affect our results beyond this re-scaling and a slight difference in interpretation.

5.2 *Instrumental variable results*

We now turn to our results when we allow for R and K and, then, for R, K, and T to be determined by supply as well as demand factors. Depending on the case, we either estimate equations (8) and (9) or equations (10), (11), and (12) in a preliminary step prior to smoothing R. As for the explanatory variables in these regressions, recall that we include urban area fixed effects, distance to the centre (with a coefficient specific to each urban areas), three municipal socioeconomic characteristics (log mean income, its standard deviation, and the share of population with a university degree), geological variables (terrain ruggedness, and classes of soil erodability, soil hydrogeology class, and soil dominant parent material), and three land use variables (share of built-up land, share of urbanised land, and share of agricultural land). In our preferred specification, we use the urban area fixed effect (after conditioning out wages in the construction industry), distance to centre (with a coefficient specific to each urban area), and municipal socioeconomic characteristics as the demand instruments. Although we do not develop a procedure to test for weak instruments in our context, there is little doubt that these variables strongly predict our potentially endogenous variables. In Combes *et al.* (2016), urban area fixed effect and (log) distance to the centre explain 63% of the variation of the price of land per square meter in the data for 2006-2012.

For our preferred instrumentation of R and T, panel (A) of table 4 reports a first series of estimations that mirror panel (A) of table 2. The results are nearly exactly the same with a similar pattern of increasing elasticity of housing production with respect to capital as higher deciles of parcel size are considered. The only difference is that the instrumented elasticities are marginally higher relative to those estimated in table 2. Even though the estimated differences in elasticity across the

Table 4: log housing production in urban area with endogenous variables, OLS by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Endogenous R and K									
$\log(K)$	0.784^a	0.786^a	0.787^a	0.789^a	0.793^a	0.799^a	0.803^a	0.807^a	0.812^a
	(0.00082)	(0.00049)	(0.00039)	(0.00051)	(0.00065)	(0.00085)	(0.0011)	(0.0012)	(0.0014)
Panel (B): Endogenous R and K									
$\log(K)$	0.383^a	1.008^a	1.510^a	1.851^a	1.967^a	1.929^a	1.602^a	1.507^a	1.664^a
	(0.101)	(0.064)	(0.054)	(0.059)	(0.072)	(0.098)	(0.126)	(0.145)	(0.160)
$[\log(K)]^2$	0.017^a	-0.009^a	-0.031^a	-0.045^a	-0.050^a	-0.048^a	-0.034^a	-0.030^a	-0.036^a
	(0.0043)	(0.0027)	(0.0023)	(0.0025)	(0.0030)	(0.0041)	(0.0053)	(0.0061)	(0.0068)
Panel (C): Endogenous R, K, and T									
$\log(K)$	0.789^a	0.798^a	0.791^a	0.790^a	0.788^a	0.784^a	0.786^a	0.798^a	0.807^a
	(0.0017)	(0.0012)	(0.0017)	(0.0016)	(0.0019)	(0.0020)	(0.0025)	(0.0021)	(0.0026)
Panel (D): Endogenous R, K, and T									
$\log(K)$	0.311	0.849^a	1.080^a	2.007^a	2.796^a	2.536^a	2.791^a	2.098^a	1.938^a
	(0.260)	(0.179)	(0.282)	(0.252)	(0.362)	(0.327)	(0.367)	(0.297)	(0.350)
$[\log(K)]^2$	0.020^c	-0.002	-0.012	-0.051^a	-0.085^a	-0.074^a	-0.085^a	-0.055^a	-0.048^a
	(0.011)	(0.0076)	(0.012)	(0.011)	(0.015)	(0.014)	(0.016)	(0.013)	(0.015)

Notes: OLS regressions with a constant in all columns. 900 observations for each regression. The R^2 is 1.00 in all specifications. Capital and parcel price are treated as endogenous and constructed using urban area fixed effects (after conditioning out construction wages), distance to the centre (urban-area specific), and income variables (log mean municipal income, log standard error of income, and share of population with a university degree). Parcel size is treated as exogenous in panels (A) and (B) and as endogenous in panels (C) and (D). Bootstrapped standard errors in parentheses. a, b, c: significant at 1%, 5%, 10%. Non-parametric estimates of housing production rely on 213,788 observations (instead of 218,657 when we do not instrument).

two tables for the same decile of parcel area in panel (A) are significant in a statistical sense, they are economically tiny since the difference is always less than 0.013 for elasticities around 0.8.

Panel (B) of table 4 adds a quadratic term for $\log K$ and duplicates the specifications of the corresponding panel of table 2. The results now indicate the presence of a mild log concavity for all deciles of parcel size except the first one. This is in contrast with table 2 where the results point towards modest log convexity. While there is some variation in the estimated coefficient on the quadratic term in $\log K$ across deciles of parcel size, the largest one in absolute value is 0.05 for the fifth decile of parcel size. With $\log K$ varying from 11.4 to 12.2 between the bottom and top decile of capital, this log concavity implies that the capital elasticity of housing production is only about 0.04 lower at the top decile relative to the bottom decile. Again, for an elasticity of about 0.8, this

Table 5: log housing production, OLS by class of urban area population with endogenous variables

City size class	Urban areas	0-50	50-100	100-200	200-500	500+
Panel (A)						
$\log(K)$	0.795^a	0.842^a	0.829^a	0.822^a	0.797^a	0.736^a
	(0.00041)	(0.00080)	(0.00084)	(0.00070)	(0.00090)	(0.053)
Panel (B)						
$\log(K)$	1.491^a	1.010^a	1.125^a	1.950^a	1.721^a	1.372^a
	(0.057)	(0.123)	(0.164)	(0.098)	(0.134)	(0.245)
$[\log(K)]^2$	-0.029^a	-0.007	-0.013^c	-0.048^a	-0.039^a	-0.027^b
	(0.0024)	(0.0052)	(0.0070)	(0.0042)	(0.0056)	(0.012)

Notes: OLS regressions with decile fixed effects in all columns. 8,100 observations for each regression. The R^2 is 1.00 in all specifications. Capital and parcel price are treated as endogenous and constructed using urban area fixed effects (after conditioning out construction wages), distance to the centre (urban-area specific), and income variables. Parcel size is treated as exogenous. Bootstrapped standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%. We cannot report results for the entire country given that our instrumentation relies on urban area fixed effects and distance to the centre, which are unavailable in rural areas. Similarly we cannot implement our approach when we consider Paris alone.

is economically small.

Before interpreting this finding further, we confirm it in a variety of ways. Panels (C) and (D) of table 4 duplicate the previous two panels but also consider that parcel size is endogenous. The results are qualitatively similar and quantitatively close. The only difference is that the increase of the capital elasticity across parcel size deciles in panel (C) is slightly less than in panel (A) and the log concavity is slightly more pronounced in panel (D) than in panel (B).

Table 5 duplicates table 3 by size class of urban areas instrumenting R and T with our preferred instruments. Using instrumented values of R and T, we estimate again marginally higher capital elasticities and some log concavity instead of log convexity in table 3.

Table 6 report results experimenting with the set of instruments. Panels (A) and (B) only include the urban area of a parcel to predict its price and capital investment. The results are qualitatively the same as those of the two panels of table 4. Nonetheless, for this blunt and rudimentary instrumentation, we note a greater dispersion of the capital elasticity in panel (A) and more log concavity, especially for the lower deciles of parcel size in panel (B). Panels (C) and (D) rely again on the urban area of a parcel to predict its price and capital investment but condition out local wages in the construction industry from the estimated urban area fixed effects. With this specification, the

Table 6: log housing production in urban with endogenous variables, OLS by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Urban area fixed effects only									
$\log(K)$	0.755^a	0.775^a	0.788^a	0.796^a	0.802^a	0.810^a	0.816^a	0.820^a	0.824^a
	(0.0017)	(0.0017)	(0.0011)	(0.00088)	(0.0011)	(0.0013)	(0.0015)	(0.0015)	(0.0019)
Panel (B): Urban area fixed effects only									
$\log(K)$	4.760^a	4.437^a	2.934^a	2.271^a	2.410^a	2.422^a	2.395^a	2.387^a	2.580^a
	(0.294)	(0.187)	(0.200)	(0.134)	(0.174)	(0.207)	(0.224)	(0.254)	(0.308)
$[\log(K)]^2$	-0.169^a	-0.155^a	-0.091^a	-0.062^a	-0.068^a	-0.068^a	-0.067^a	-0.066^a	-0.074^a
	(0.012)	(0.0080)	(0.0085)	(0.0056)	(0.0074)	(0.0088)	(0.0095)	(0.011)	(0.013)
Panel (C): Urban area fixed effects net of construction wages									
$\log(K)$	0.789^a	0.780^a	0.776^a	0.775^a	0.776^a	0.778^a	0.779^a	0.781^a	0.787^a
	(0.0026)	(0.00080)	(0.00065)	(0.00074)	(0.00093)	(0.0015)	(0.0022)	(0.0028)	(0.0045)
Panel (D): Urban area fixed effects net of construction wages									
$\log(K)$	-0.523	-0.126	0.824^a	1.244^a	1.385^a	1.061^a	0.194	-0.174	0.661
	(0.441)	(0.150)	(0.138)	(0.115)	(0.172)	(0.324)	(0.480)	(0.645)	(0.948)
$[\log(K)]^2$	0.055^a	0.038^a	-0.002^a	-0.020^a	-0.026^a	-0.012	0.025	0.040	0.005
	(0.019)	(0.0063)	(0.0058)	(0.0049)	(0.0073)	(0.014)	(0.020)	(0.027)	(0.040)
Panel (E): Urban area fixed effects, distance effects, income, and land use									
$\log(K)$	0.761^a	0.775^a	0.782^a	0.788^a	0.795^a	0.803^a	0.808^a	0.814^a	0.821^a
	(0.00096)	(0.00064)	(0.00051)	(0.00056)	(0.00081)	(0.0010)	(0.0011)	(0.0014)	(0.0017)
Panel (F): Urban area fixed effects, distance effects, income, and land use									
$\log(K)$	3.209^a	2.958^a	2.919^a	3.045^a	3.194^a	3.270^a	3.332^a	3.258^a	3.175^a
	(0.066)	(0.059)	(0.054)	(0.057)	(0.060)	(0.069)	(0.076)	(0.106)	(0.123)
$[\log(K)]^2$	-0.103^a	-0.092^a	-0.090^a	-0.095^a	-0.101^a	-0.104^a	-0.107^a	-0.103^a	-0.099^a
	(0.0028)	(0.0025)	(0.0023)	(0.0024)	(0.0026)	(0.0029)	(0.0032)	(0.0045)	(0.0053)
Panel (G): ——— net of construction wages with T endogenous									
$\log(K)$	0.779^a	0.786^a	0.790^a	0.787^a	0.790^a	0.793^a	0.792^a	0.805^a	0.813^a
	(0.0014)	(0.0012)	(0.0014)	(0.0016)	(0.0011)	(0.0016)	(0.0021)	(0.0022)	(0.0024)
Panel (H): ——— net of construction wages with T endogenous									
$\log(K)$	-0.270	1.192^a	2.409^a	2.741^a	2.668^a	2.152^a	2.418^a	1.930^a	1.426^a
	(0.240)	(0.202)	(0.229)	(0.222)	(0.195)	(0.167)	(0.230)	(0.276)	(0.290)
$[\log(K)]^2$	0.044^a	-0.017^b	-0.068^a	-0.082^a	-0.079^a	-0.057^a	-0.069^a	-0.047^a	-0.026^b
	(0.010)	(0.0085)	(0.0097)	(0.0094)	(0.0083)	(0.0071)	(0.0098)	(0.012)	(0.012)

Notes: OLS regressions with a constant in all columns. In all panels (E)-(H), distance to the centre is urban-area specific; income variables are log mean municipal income, log standard error of income, and share of population with a university degree; geology variables are ruggedness, soil erodability, soil hydrogeological class, dominant parent material for two main classes of (lighter) soils; land use variables are three land use variables share of built-up land , share of urbanised land, and share of agricultural land. 900 observations for each regression. The R^2 is 1.00 in all specifications. Robust standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

differences in capital elasticity across parcel size deciles are minimal. Depending on the deciles, the production function of housing is either mildly log concave or mildly log convex. Panels (E) and (F) include urban area fixed effects, distance to the centre (with an effect specific to each urban area), income, and land-use variables among the demand determinants but do not condition out construction wages. Finally, panels (G) and (H) additionally condition out construction wages from urban area fixed effects and instrument parcel size. Although, one may (rightfully) object that land use patterns may reflect more than just local demand conditions (even after conditioning out geological characteristics), the similarity with the results of table 4 shows that our instrumented results are not sensitive to the exact details of what we include in our set of instruments.

Overall our instrumented results suggest a marginally higher elasticity of housing production with respect to capital. The difference is nonetheless too small to be economically meaningful. More importantly, our instrumented results indicate that the production function for housing is mildly log concave rather than log convex when we do not instrument. This change makes intuitive sense in relation to the possible biases described above. For parcels of the same area, we expect parcels that are more difficult to built to require more capital. The price of these parcels will also be lower due to this. With lower prices for parcels receiving a greater capital investment, the share of capital will thus increase with the amount of capital used to build the house (all else equal). This can bias our results and generate an apparent log convexity of the production of housing when we do not instrument.

6. Recovering a functional form

So far, we have non-parametrically estimated the production of housing as a function of capital, given parcel size. We then estimated simple regressions to assess the shape of this non-parametric function. As a first approximation, the production function of housing is log linear with a share of capital of about 0.80. However, a more detailed look suggests mild log convexity when using the raw data and, perhaps more reasonably, modest log concavity with instrumented values.

In this section, we asses a variety of functional forms for the production function of housing. Given our results, measuring the goodness of fit of different specifications is unlikely to be informative since the simplest log linear specifications always have an R^2 close to unity. Instead, we duplicate our estimation of the capital elasticity of housing production for each decile of parcel size, having imposed specific functional forms for the production of housing instead of doing it non-parametrically. We can then compare the results we obtain using these (pre-imposed) functional forms to our earlier, non-parametric estimations results.

For the exposition to remain concrete, consider a CES production function. The production of housing is given by $H = A\left(\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)T^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)}$ where σ is the elasticity of substitution between land and capital and A is a productivity shifter. Using equation (4) and the partial derivative of the CES production function with respect to K, we obtain the following cost share:

$$\frac{rK^*}{rK^* + R(K^*,T)} = \frac{\alpha(K^*)^{1-1/\sigma}}{\alpha(K^*)^{1-1/\sigma} + (1-\alpha)T^{1-1/\sigma}}\,. \tag{13}$$

From transactions data, we can estimate α and σ using equation (13) by minimising the sum of the squared distances between actual costs shares and those predicted by a CES production function.[34] We then compute "CES productions" of housing at the points of our grid using the estimated parameters and perform the same regression as in table 2. We also repeat the same exercise using instrumented values as in table 4. Aside from the CES, we also make an assessment for the Cobb-Douglas and for the second- and third-order translog production functions.

Table 7 reports a first series of non-instrumented results. In panel (A) of table 7, we can see that imposing Cobb-Douglas to the data leads to about the same capital elasticities of housing production as in table 2 but, by construction, this fails to replicate the higher elasticity for larger parcels. By construction again, the capital elasticity of 0.776 that we recover is the same as the one estimated from factor shares. Panel (B) also includes a quadratic term for $\log K$ and estimates, as expected, the same coefficient of 0.776 for $\log K$ and a coefficient of 0 (or extremely close to) for its square.

For the CES case, we note that the estimated parameter values for the production function are $\alpha = 0.794$ and $\sigma = 0.902$. This value of σ close to one implies a situation close to the Cobb-Douglas case. In panel (C), the coefficients on $\log K$ are, unsurprisingly, very close to those estimated in the Cobb-Douglas case except that this panel also replicates the tendency of the capital elasticity to increase with parcel size. In panel (D), we estimate minimal log concavity in capital instead of the log convexity reported in panel (B) of table 2. In panel (E), the second-order translog is also able to replicate the upward trend in the capital elasticity. These elasticities are about equal to those reported in table 2. In panel (F), the coefficients on the quadratic term in log capital reproduce,

[34] We weigh observations with the kernel weights used for smoothing to take into account the distribution of observations in the data.

Table 7: log housing production fitting specific functional forms, OLS by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Cobb-Douglas									
$\log(K)$	0.776^a	0.776^a	0.776^a	0.776^a	0.776^a	0.776^a	0.776^a	0.776^a	0.776^a
	(0.00054)	(0.00054)	(0.00054)	(0.00054)	(0.00054)	(0.00054)	(0.00054)	(0.00054)	(0.00054)
Panel (B): Cobb-Douglas									
$\log(K)$	0.776^a	0.776^a	0.776^a	0.776^a	0.776^a	0.776^a	0.776^a	0.776^a	0.776^a
	(0.00054)	(0.00054)	(0.00054)	(0.00054)	(0.00054)	(0.00054)	(0.00054)	(0.00054)	(0.00054)
$[\log(K)]^2$	2.17e-7	-1.45e-7	-1e-7	-1.56e-7	-1.62e-7	-1.08e-7	-2.02e-7	-0.99e-7	-1.25e-7
	(1.7e-7)	(1.5e-7)	(1.5e-7)	(1.3e-7)	(1.5e-7)	(1.4e-7)	(1.5e-7)	(1.5e-7)	(1.6e-7)
Panel (C): CES									
$\log(K)$	0.762^a	0.767^a	0.770^a	0.773^a	0.776^a	0.778^a	0.780^a	0.781^a	0.783^a
	(0.00057)	(0.00057)	(0.00057)	(0.00057)	(0.00056)	(0.00056)	(0.00056)	(0.00056)	(0.00056)
Panel (D): CES									
$\log(K)$	0.928^a	0.935^a	0.939^a	0.943^a	0.946^a	0.948^a	0.951^a	0.952^a	0.954^a
	(0.00084)	(0.00085)	(0.00085)	(0.00086)	(0.00086)	(0.00086)	(0.00086)	(0.00087)	(0.00087)
$[\log(K)]^2$	-0.0070^a	-0.0070^a	-0.0071^a	-0.0071^a	-0.0072^a	-0.0072^a	-0.0072^a	-0.0072^a	-0.0072^a
	(0.00003)	(0.00003)	(0.00003)	(0.00003)	(0.00003)	(0.00003)	(0.00003)	(0.00003)	(0.00003)
Panel (E): Second-order translog									
$\log(K)$	0.771^a	0.775^a	0.778^a	0.781^a	0.783^a	0.785^a	0.786^a	0.788^a	0.789^a
	(0.00072)	(0.00057)	(0.00053)	(0.00054)	(0.00059)	(0.00065)	(0.00071)	(0.00077)	(0.00083)
Panel (F): Second-order translog									
$\log(K)$	0.184^a	0.188^a	0.191^a	0.193^a	0.195^a	0.197^a	0.199^a	0.200^a	0.201^a
	(0.015)	(0.015)	(0.015)	(0.015)	(0.015)	(0.016)	(0.016)	(0.016)	(0.016)
$[\log(K)]^2$	0.0247^a	0.0247^a	0.0247^a	0.0247^a	0.0247^a	0.0247^a	0.0247^a	0.0247^a	0.0247^a
	(0.00065)	(0.00065)	(0.00065)	(0.00065)	(0.00065)	(0.00065)	(0.00065)	(0.00065)	(0.00065)
Panel (G): Third-order translog									
$\log(K)$	0.775^a	0.777^a	0.779^a	0.781^a	0.784^a	0.787^a	0.789^a	0.792^a	0.794^a
	(0.00084)	(0.00060)	(0.00063)	(0.00060)	(0.00058)	(0.00067)	(0.00087)	(0.0011)	(0.0014)
Panel (H): Third-order translog									
$\log(K)$	0.257^a	0.272^a	0.284^a	0.295^a	0.304^a	0.312^a	0.320^a	0.327^a	0.333^a
	(0.026)	(0.018)	(0.016)	(0.017)	(0.022)	(0.026)	(0.030)	(0.034)	(0.037)
$[\log(K)]^2$	0.0218^a	0.0212^a	0.0208^a	0.0205^a	0.0202^a	0.0200^a	0.0198^a	0.0196^a	0.0194^a
	(0.0011)	(0.00076)	(0.00069)	(0.00078)	(0.00094)	(0.0011)	(0.0013)	(0.0014)	(0.0016)

Notes: OLS regressions with a constant in all columns. 900 observations for each regression. The R^2 is 1.00 in all specifications. Bootstrapped standard errors in parentheses. a, b, c: significant at 1%, 5%, 10%. For the second-order translog, there is a single coefficient for all deciles of parcel size for the term in log K squared by definition.

albeit more strongly, the log convexity in capital estimated in table 2. Panels (G) and (H) report results for a third-order translog function instead of a second-order translog. The results match those obtained in table 2 marginally better than those of the second-order translog.

Table 8 repeats the same exercise for instrumented values of K and R instead of the observed values used in table 7. The results should now be compared with those of table 4. Again, the Cobb-Douglas specification delivers capital elasticities of the right magnitude but is obviously unable to generate the log concavity estimated in panel (B) of table 4. The CES specification is able to reproduce both the tendency of the capital elasticity to be higher in higher deciles of parcel size in panel (C) and the estimated log concavity of H in K when adding a quadratic term in panel (D). The elasticity of substitution between land and capital estimated from the data is 0.795, which is somewhat different from the elasticity of 0.902 estimated above when not instrumenting. The amount of concavity obtained from the CES specification is tiny and slightly less than what is estimated in table 4. The two translog specifications in panels (E)-(H) can match closely the capital elasticities estimated in table 4 and replicate the log concavity of H in K. For the specification that includes squared log capital, the third-order translog of panel (H) offers the best match with the instrumented results of table 4.

We draw three conclusions from this section. First, we confirm that a Cobb-Douglas specification provides a good first-order description of the data. Second, and consistent with this, we find that the estimation of a CES production function for housing has an elasticity of substitution between land and capital inputs of 0.8 to 0.9 depending on whether we instrument. The CES provides a better description of the instrumented data but the gain relative to a Cobb-Douglas specification is small. Overall, the third-order translog offers the closest approximation to our non-parametric results but the gain from this more flexible functional form is again small. Third, none of the functional form we consider is able to match exactly the results of our non-parametric estimation. Put differently, these results suggest that it is better to use a non-parametric approach and then provide an functional form approximation than impose a functional form directly into the estimation.

Table 8: log housing production fitting specific functional forms and using instrumented values, OLS by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): Cobb-Douglas									
$\log(K)$	0.789^a	0.789^a	0.789^a	0.789^a	0.789^a	0.789^a	0.789^a	0.789^a	0.789^a
	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)
Panel (B): Cobb-Douglas									
$\log(K)$	0.789^a	0.789^a	0.789^a	0.789^a	0.789^a	0.789^a	0.789^a	0.789^a	0.789^a
	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)	(0.00031)
$[\log(K)]^2$	2.60e-7	6.83e-7	-1.67e-7	-3.19e-7	0.04e-7	0.82e-7	1.09e-7	2.75e-7	-1.75e-7
	(5.3e-7)	(5.7e-7)	(6.0e-7)	(5.5e-7)	(5.7e-7)	(5.7e-7)	(5.6e-7)	(5.3e-7)	(6.0e-7)
Panel (C): CES									
$\log(K)$	0.779^a	0.785^a	0.789^a	0.792^a	0.795^a	0.797^a	0.799^a	0.800^a	0.802^a
	(0.00040)	(0.00041)	(0.00041)	(0.00042)	(0.00043)	(0.00043)	(0.00044)	(0.00044)	(0.00044)
Panel (D): CES									
$\log(K)$	0.958^a	0.965^a	0.969^a	0.973^a	0.976^a	0.978^a	0.981^a	0.982^a	0.984^a
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0011)
$[\log(K)]^2$	-0.00757^a	-0.00760^a	-0.00763^a	-0.00765^a	-0.00766^a	-0.00768^a	-0.00769^a	-0.00770^a	-0.00771^a
	(0.00004)	(0.00004)	(0.00004)	(0.00003)	(0.00003)	(0.00003)	(0.00003)	(0.00003)	(0.00003)
Panel (E): Second-order translog									
$\log(K)$	0.776^a	0.783^a	0.787^a	0.791^a	0.794^a	0.797^a	0.799^a	0.801^a	0.803^a
	(0.00078)	(0.00048)	(0.00034)	(0.00035)	(0.00044)	(0.00055)	(0.00066)	(0.00076)	(0.00085)
Panel (F): Second-order translog									
$\log(K)$	1.479^a	1.485^a	1.490^a	1.493^a	1.497^a	1.499^a	1.502^a	1.504^a	1.506^a
	(0.041)	(0.041)	(0.042)	(0.042)	(0.042)	(0.042)	(0.042)	(0.042)	(0.042)
$[\log(K)]^2$	-0.0297^a	-0.0297^a	-0.0297^a	-0.0297^a	-0.0297^a	-0.0297^a	-0.0297^a	-0.0297^a	-0.0297^a
	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)
Panel (G): Third-order translog									
$\log(K)$	0.785^a	0.785^a	0.787^a	0.790^a	0.795^a	0.799^a	0.803^a	0.808^a	0.812^a
	(0.00098)	(0.00048)	(0.00041)	(0.00039)	(0.00044)	(0.00059)	(0.00083)	(0.0011)	(0.0014)
Panel (H): Third-order translog									
$\log(K)$	0.737^a	1.137^a	1.439^a	1.680^a	1.882^a	2.056^a	2.209^a	2.345^a	2.468^a
	(0.089)	(0.055)	(0.041)	(0.045)	(0.057)	(0.071)	(0.084)	(0.097)	(0.108)
$[\log(K)]^2$	0.00203	-0.0149^a	-0.0276^a	-0.0376^a	-0.0460^a	-0.0531^a	-0.0594^a	-0.0650^a	-0.0700^a
	(0.0038)	(0.0023)	(0.0017)	(0.0019)	(0.0024)	(0.0030)	(0.0035)	(0.0041)	(0.0046)

Notes: OLS regressions with a constant in all columns. Capital and parcel price are treated as endogenous as in table 4. Parcel size is treated as exogenous. 900 observations for each regression. The R^2 is 1.00 in all specifications. Bootstrapped standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 9: Explaining the price of land per square metre

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable	Raw data						Smoothed data	
Log parcel size	-0.991[a]	-0.971[a]	-0.759[a]	-0.644[a]	-0.656[a]	0.922[a]	-1.084[a]	-0.254[a]
	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.024)	(0.001)	(0.063)
Log parcel size squared						-0.114[a]		-0.060[a]
						(0.002)		(0.005)
Parcel controls	No	Yes	Yes	Yes	Yes	Yes	No	No
Urban area indicator	No	No	Yes	Yes	Yes	Yes	No	No
Distance to the centre	No	No	No	Yes	Yes	Yes	No	No
Municipal controls	No	No	No	No	Yes	Yes	No	No
Observations	213,788	213,788	213,788	213,788	213,788	213,788	90,000	90,000
R^2	0.43	0.45	0.74	0.79	0.80	0.81	0.81	0.81

Note OLS regressions with year effects in all columns. [a]: significant at 1% level; [b]: significant at 5% level; [a]: significant at 10% level. Parcel controls include indicator variables for whether the parcel is serviced and three types of intermediaries through whom the parcel may have been bought. Municipal controls include log area, log mean income of the year, log standard error of income of the year, share of municipal land that is urbanised (covered) in 2006, share of municipal land for agriculture, ruggedness, soil erodability, soil hydrogeological class, dominant parent material for two main classes of (lighter) soils.

7. Full identification?

In our approach so far, we have assumed that parcels were taken as given by house builders. We think that taking parcels as exogenous is reasonable in the French institutional context. Recall that our data pertain to single-family homes built mostly individually (or in small numbers) as in-fills. This said, it is easy to expand the model-based approach developed in section 2 and allow for house builders to maximise their profits with respect to both capital and parcel size. This extension is fully developed in Appendix C.

We assume that land rent is linear in land area $R = \widetilde{R}(x) \times T$, in which case the unit price of land is constant at a given location regardless of parcel size. This assumption is natural in a context of divisible land and competitive land-owners. It prevents any arbitrage gain from re-selling part of parcels that would have been bought at lower unit prices. A second argument in the profit function of house builders introduces a second first-order condition. Importantly, for the two first-order conditions of the builder's problem to be consistent with zero profit, the housing production function is necessarily constant returns to scale.

Substituting both first-order conditions into the zero profit condition allows us to fully identify the production function of housing. This is in contrast with our partially identified approach so far. To assess whether imposing constant returns to scale is warranted, we perform two checks.

First, we regress the log of the price of parcels per unit of land $\widetilde{R}(x)$ on log parcel size and other parcel characteristics. The results are reported table 9. Column 1 regresses the log price of parcels per square metre on the log of their size. Strikingly, the coefficient is about minus one. Adding parcel controls, urban area indicators, distance to the centre (with a coefficient specific to each urban area), and many municipal controls in columns 2 to 5 lowers the magnitude of the coefficient on log parcel size. Nonetheless, even with a full set of controls, the coefficient on parcel size remains large in magnitude at about -0.66.[35] Adding a quadratic term on log parcel size in column 6 provides evidence of some log concavity indicating that the marginal price of land declines faster for larger parcels. Columns 7 and 8 use kernel-smoothed land price data instead of the actual transaction price data used in columns 1 to 6. The results in these last two columns essentially confirm the result that unit land prices strongly decline with parcel size. When builders are able to maximise profits with respect to parcel size, the unit price of land should be equalised across parcels of different sizes. Our results clearly reject this prediction.

For our second exercise, we proceed in the spirit of what we do in section 6 and re-estimate housing production under the added restriction of constant returns to scale. We can then regress the constant-return values of $\log H$ on $\log K$. If imposing constant returns to scale was innocuous, we should find results similar to those obtained above under partial identification when this assumption is not imposed. The results are reported in table 10. The first two panels of this table are analogous to those of tables 2 while the last two instrumented panels are analogous to the two panels of table 4.

For first decile of parcel sizes, the results from panels (A) and (B) of table 10 are similar to those of table 2. For subsequent deciles, the capital elasticity falls from 0.768 to 0.712 in table 10 while this elasticity increases in table 2. We observe a similar pattern of divergence, albeit in the opposite direction, in the last two panels of table 10 relative the two corresponding panels of table 4 when instrumenting. We interpret this divergence between the results obtained with a constant-return assumption and those obtained without as evidence that imposing constant returns to scale may be warranted when considering small parcels but becomes increasingly less appropriate when we consider larger parcels. This interpretation is consistent with the results of table 9 showing that the price of land per square metre declines with parcel size. In turn, parcels are probably best

[35]For a very similar specification using US land price data, Albouy and Ehrlich (2013) estimate a coefficient of -0.61.

Table 10: log housing production imposing constant returns to scale in production, OLS by size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): raw data									
$\log(K)$	0.768^a	0.755^a	0.743^a	0.733^a	0.726^a	0.721^a	0.717^a	0.715^a	0.712^a
	(0.00073)	(0.00090)	(0.0012)	(0.0014)	(0.0015)	(0.0017)	(0.0019)	(0.0021)	(0.0024)
Panel (B): raw data									
$\log(K)$	0.378^a	-0.215^a	-0.702^a	-1.207^a	-1.735^a	-2.237^a	-2.713^a	-3.121^a	-3.450^a
	(0.028)	(0.058)	(0.081)	(0.096)	(0.111)	(0.130)	(0.154)	(0.178)	(0.197)
$[\log(K)]^2$	0.016^a	0.041^a	0.061^a	0.082^a	0.104^a	0.125^a	0.144^a	0.162^a	0.175^a
	(0.0012)	(0.0024)	(0.0034)	(0.0041)	(0.0047)	(0.0055)	(0.0065)	(0.0076)	(0.0083)
Panel (C): instrumented data									
$\log(K)$	0.784^a	0.781^a	0.789^a	0.802^a	0.817^a	0.830^a	0.841^a	0.849^a	0.856^a
	(0.00081)	(0.0019)	(0.0026)	(0.0029)	(0.0032)	(0.0037)	(0.0041)	(0.0046)	(0.0050)
Panel (D): instrumented data									
$\log(K)$	0.383^a	1.049^a	1.370^a	1.033^a	0.215	-0.897^a	-2.239^a	-3.621^a	-4.956^a
	(0.103)	(0.340)	(0.449)	(0.502)	(0.548)	(0.603)	(0.669)	(0.749)	(0.826)
$[\log(K)]^2$	0.017^a	-0.011^a	-0.025^a	-0.010*	0.025^a	0.073^a	0.130^a	0.189^a	0.246^a
	(0.0044)	(0.014)	(0.019)	(0.021)	(0.023)	(0.026)	(0.028)	(0.031)	(0.035)

Notes: OLS regressions with a constant in all columns. 300 observations for each regression. In panels (c) and (d), capital and parcel price are treated as endogenous as in table 4. The R^2 is 1.00 in all specifications. Bootstrapped standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

viewed as exogenous because of their indivisibility rather than the product of a maximising choice by house builders. This said, our rejection of constant returns to scale is like our rejection of the Cobb-Douglas functional form. Although, we can formally reject that houses are produced under constant returns, this remains a reasonable first-order approximation.

8. Conclusions

We develop a novel approach to estimate the production function of housing. Our approach relies on the notion that, although heterogeneous in many dimensions, houses all provide housing services. The price of a house is then the product of the price of housing per unit (which varies across locations) and the number of housing units provided by this house. To separate these two quantities, we assume that housing is competitively provided. Then, the first-order condition for house builders determines the marginal *value product* of capital. Using the zero-profit condition, we can eliminate the price of housing per unit from the first-order condition and isolate the marginal *product* of capital investment when building a house. For parcels of a given size, we essentially sum this marginal product across houses in different locations that have optimally received different levels of capital and recover the production of housing associated with each level of capital. Although our approach could potentially be applied to other production function estimations, we believe that using it for housing is particularly appropriate because we can rely on the large spatial variations of land prices, a fundamentally important input in our context.

Our main results are that the production function of housing is reasonably well approximated by a Cobb-Douglas production function under constant returns. This said, we can nonetheless show that this is not exactly true. Our preferred results indicate a mild amount of log concavity in the production function of housing and an elasticity of housing production with respect to capital increasing with parcel size, which is consistent with a log super-modular function.

There are three challenges that future work will need to deal with. First, the production function of housing we estimate is arguably affected by land use regulations and the building code. Obtaining information about the regulations that apply to each parcel and changes in the building code together with plausible sources of exogenous variation for these will be necessary to assess the effects of land use regulations and of the building code on the efficiency of construction. Second, we implicitly assume that housing is perfectly divisible (unlike parcels). We do not expect households who purchase a new house to get exactly the quantity of housing they wanted. In turn, the willingness to pay of a household for a unit of housing may decline as the house they consider deviates from their preferred choice. Exploring the implications of the indivisibility of housing in our framework is a natural next step. Finally, we assume an integrated housing market. While this may be a reasonable assumption for new houses in a given city or for buyers that belong to the same occupational group, it will be important to consider richer forms of heterogeneity in the demand for housing.

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Appendix A. Other data

Urban areas. We use the 1999 delineation of urban areas from the French statistical institute (INSEE).

Wages. We construct measures of wages for blue collar workers in the construction industry for all French urban areas from the French labour force administrative records (DADS - Déclarations Annuelles des Données Sociales).

Education. We construct measures of the share of population with a college or university degree for all French municipalities from the French census for 2006. We consider all higher education degrees that sanction two years of study or more after high school.

Income. Mean household income and its standard deviation by municipality can be constructed using information from each cadastral section (about 100 housing units on average) contained in the FILOCOM repository. This repository is managed by the *Direction Générale des Finances Publiques* of the French Ministry of Finance. It contains a record of all housing units and their occupants which they match to income tax records.

Soil variables We use the European Soil Database compiled by the European Soil Data Centre. The data originally come as a raster data file with cells of 1 km per 1 km. We aggregated it at the level of each municipality and urban area. We refer to Combes, Duranton, Gobillon, and Roux (2010) for further description of these data.

Land use. We compute the fraction of land that is built up in each municipality using information from *BD Topo* (version 2.1) from the French National Geographical Institute. This data set is originally produced using satellite imagery combined with the French land registry. It reports information for more than 95% of buildings in the country including their footprint, height, and use with an accuracy of one metre. We also use information from the *Corine Land Cover* dataset to compute the share of agricultural and developed land in each municipality.

Appendix B. Supplementary results

Table 11 report results for different levels of completion, table 12 reports result by occupational groups of buyers, table 13 reports results for different smoothing bandwidth, and table 14 reports results when we do not apply our user cost correction.

Table 11: log housing production in urban areas at various degrees of completion, OLS by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): fully finished units									
$\log(K)$	0.784^a	0.791^a	0.792^a	0.790^a	0.793^a	0.796^a	0.800^a	0.802^a	0.795^a
	(0.0011)	(0.0010)	(0.0011)	(0.0013)	(0.0016)	(0.0019)	(0.0020)	(0.0027)	(0.0033)
Panel (B): ready to decorate									
$\log(K)$	0.766^a	0.775^a	0.777^a	0.777^a	0.779^a	0.786^a	0.789^a	0.791^a	0.796^a
	(0.00084)	(0.00071)	(0.00077)	(0.00072)	(0.00092)	(0.0012)	(0.0013)	(0.0016)	(0.0028)
Panel (C): structure completed									
$\log(K)$	0.745^a	0.752^a	0.756^a	0.755^a	0.755^a	0.758^a	0.761^a	0.761^a	0.758^a
	(0.0025)	(0.0021)	(0.0021)	(0.0028)	(0.0031)	(0.0034)	(0.0043)	(0.052)	(0.067)

Notes: OLS regressions with a constant in all columns. Bootstrapped standard errors in parentheses. 900 observations for each regression. The R^2 is 1.00 in all specifications. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 12: log housing production in urban areas across owners' occupations, OLS by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): executives									
$\log(K)$	0.759^a	0.768^a	0.771^a	0.769^a	0.771^a	0.772^a	0.772^a	0.774^a	0.765^a
	(0.0013)	(0.0013)	(0.0011)	(0.013)	(0.0015)	(0.0020)	(0.0028)	(0.0034)	(0.0045)
Panel (B): intermediate occupations									
$\log(K)$	0.770^a	0.778^a	0.779^a	0.780^a	0.781^a	0.784^a	0.790^a	0.793^a	0.788^a
	(0.0021)	(0.0020)	(0.0019)	(0.0019)	(0.0023)	(0.0030)	(0.0031)	(0.0033)	(0.0055)
Panel (C): clerical and blue-collar workers									
$\log(K)$	0.781^a	0.784^a	0.785^a	0.787^a	0.791^a	0.794^a	0.797^a	0.801^a	0.805^a
	(0.00090)	(0.00081)	(0.00084)	(0.00087)	(0.00094)	(0.0012)	(0.0014)	(0.015)	(0.018)

Notes: OLS regressions with a constant in all columns. Bootstrapped standard errors in parentheses. 900 observations for each regression. The R^2 is 1.00 in all specifications. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 13: log housing production with different smoothing bandwidth, OLS by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A): bandwidth = 0.5× rule-of-thumb bandwidth									
$\log(K)$	0.764^a	0.780^a	0.778^a	0.779^a	0.785^a	0.788^a	0.789^a	0.797^a	0.798^a
	(0.00011)	(0.00016)	(0.00019)	(0.00015)	(0.00019)	(0.00016)	(0.00013)	(0.00014)	(0.00012)
Panel (B): bandwidth = 0.5× rule-of-thumb bandwidth									
$\log(K)$	0.464^a	0.315^a	0.219^a	0.365^a	0.283^a	0.396^a	0.568^a	0.417^a	0.630^a
	(0.0045)	(0.0042)	(0.0050)	(0.0076)	(0.0091)	(0.0090)	(0.011)	(0.0060)	(0.011)
$[\log(K)]^2$	0.013^a	0.020^a	0.024^a	0.017^a	0.021^a	0.017^a	0.009^a	0.016^a	0.007^a
	(0.00019)	(0.00018)	(0.00021)	(0.00032)	(0.00038)	(0.00038)	(0.00044)	(0.00025)	(0.00048)
Panel (C): bandwidth = 0.25× rule-of-thumb bandwidth									
$\log(K)$	0.763^a	0.779^a	0.776^a	0.780^a	0.788^a	0.790^a	0.788^a	0.797^a	0.799^a
	(0.00010)	(0.00016)	(0.00019)	(0.00014)	(0.00020)	(0.00015)	(0.00013)	(0.00019)	(0.00012)
Panel (D): bandwidth = 0.25× rule-of-thumb bandwidth									
$\log(K)$	0.487^a	0.293^a	0.240^a	0.437^a	0.244^a	0.442^a	0.558^a	0.242^a	0.710^a
	(0.0045)	(0.0043)	(0.0063)	(0.0080)	(0.010)	(0.0092)	(0.011)	(0.0044)	(0.011)
$[\log(K)]^2$	0.012^a	0.020^a	0.023^a	0.014^a	0.023^a	0.015^a	0.010^a	0.023^a	0.004^a
	(0.00019)	(0.00018)	(0.00026)	(0.00034)	(0.00043)	(0.00039)	(0.00046)	(0.00018)	(0.00048)
Panel (E): bandwidth = 0.1× rule-of-thumb bandwidth									
$\log(K)$	0.766^a	0.777^a	0.779^a	0.787^a	0.790^a	0.793^a	0.787^a	0.797^a	0.802^a
	(0.00011)	(0.00017)	(0.00020)	(0.00013)	(0.00021)	(0.00016)	(0.00016)	(0.00022)	(0.00014)
Panel (F): bandwidth = 0.1× rule-of-thumb bandwidth									
$\log(K)$	0.485^a	0.279^a	0.227^a	0.532^a	0.237^a	0.420^a	0.439^a	0.152^a	1.078^a
	(0.0064)	(0.0052)	(0.0090)	(0.011)	(0.011)	(0.010)	(0.012)	(0.0073)	(0.010)
$[\log(K)]^2$	0.012^a	0.021^a	0.023^a	0.011^a	0.023^a	0.016^a	0.015^a	0.027^a	-0.012^a
	(0.00027)	(0.00022)	(0.00038)	(0.00045)	(0.00046)	(0.00042)	(0.00050)	(0.00031)	(0.00044)

Notes: OLS regressions with a constant in all columns. 900 observations for each regression. The R^2 is 1.00 in all specifications. Bootstrapped standard errors in parentheses. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Table 14: log housing production in urban areas without user cost correction, OLS by parcel size decile

Decile	1	2	3	4	5	6	7	8	9
Panel (A)									
$\log(K)$	0.624^a	0.637^a	0.639^a	0.638^a	0.642^a	0.650^a	0.653^a	0.659^a	0.661^a
	(0.00085)	(0.00071)	(0.00086)	(0.00089)	(0.0011)	(0.0015)	(0.0017)	(0.0022)	(0.0028)
Panel (B)									
$\log(K)$	0.114^a	-0.023	-0.112^a	-0.033	-0.016	0.085	0.266^a	0.232^a	0.257^a
	(0.037)	(0.029)	(0.035)	(0.042)	(0.054)	(0.070)	(0.095)	(0.115)	(0.119)
$[\log(K)]^2$	0.021^a	0.028^a	0.032^a	0.028^a	0.028^a	0.024^a	0.016^a	0.018^a	0.017^a
	(0.0016)	(0.0012)	(0.0015)	(0.0018)	(0.0023)	(0.0030)	(0.0040)	(0.0049)	(0.0050)

Notes: OLS regressions with a constant in all columns. Bootstrapped standard errors in parentheses. 900 observations for each regression. The R^2 is 1.00 in all specifications. *a*, *b*, *c*: significant at 1%, 5%, 10%.

Appendix C. Full identification: constant returns to scales

We now turn to the full identification of the housing production function (up to a constant). Note first that since builders develop a parcel of a given size at a given location, the land cost entering their profit maximization program depends only on the location x and parcel area T. We make the assumption that price of parcels is linear in their size: $R = \widetilde{R}(x)\,T$. This is consistent with the intuition that, if parcels are divisible, there should be no arbitrage possibility between parcels of different sizes.

Builders' profit at location x is $\pi = P(x)H(K,T) - rK - \widetilde{R}(x)\,T$, which is now maximised over both K and T. Aside from the first-order condition for profit maximisation with respect to capital (1), there is also one for land:

$$P(x)\,\frac{\partial H(K,T)}{\partial T} = \widetilde{R}(x)\,. \tag{C1}$$

Plugging the two first-order conditions into the zero-profit condition and simplifying by $P(x)$ leads to:

$$H(K,T) = K\,\frac{\partial H(K,T)}{\partial K} + T\,\frac{\partial H(K,T)}{\partial T}\,. \tag{C2}$$

This is Euler's condition that characterises homogeneous functions of degree 1. It implies that $H(K,T)$ exhibits constant returns to scale.

The first-order condition with respect to K, equation (1), still shows that the housing price can be rewritten as a function of K and T only, and, as before, the free-entry condition then implies that it is also the case for the total land price, $R(K,T)$. We can substitute away $P(x)$ from the free-entry condition by using now the first-order condition for T given by equation (C1). Recalling that $\widetilde{R}(x) = R(K,T)\,/T$, this leads to:

$$\frac{\partial H(K,T)}{\partial T} = \frac{\widetilde{R}(x)}{P(x)} = \frac{H(K,T)}{rK + R(K,T)}\frac{R(K,T)}{T}\,, \tag{C3}$$

which is equivalent to:

$$\frac{\partial \log H(K,T)}{\partial \log T} = \frac{R(K,T)}{rK + R(K,T)}\,. \tag{C4}$$

We obtain an expression that mirrors equation (4) for the elasticity of housing production with respect to structure. To derive the production function, we substitute expression (5) into (C4) and obtain:

$$\frac{R(K,T)}{rK + R(K,T)} = \frac{\partial \log Z(T)}{\partial \log T} - \int_K \frac{r\frac{\partial R(K,T)}{\partial T}}{\left[rK + R(K,T)\right]^2} d\log K\,. \tag{C5}$$

Integrating this equation with respect to $\log T$ yields:

$$\log Z(T) = C + \int\limits_{T} \frac{R(K,T)}{rK + R(K,T)} d\log T + \int\limits_{T}\int\limits_{K} \frac{r\frac{\partial R(K,T)}{\partial T}}{\left[rK + R(K,T)\right]^2} d\log K d\log T\,, \tag{c6}$$

where C is a constant. Substituting equation (c6) into (5), we get:

$$\begin{aligned}\log H(K,T) \quad = \quad & C + \int\limits_{T} \frac{R(K,T)}{rK + R(K,T)} d\log T + \int\limits_{K} \frac{rK}{rK + R(K,T)} d\log K \\ & + \int\limits_{T}\int\limits_{K} \frac{r\frac{\partial R(K,T)}{\partial T}}{\left[rK + R(K,T)\right]^2} d\log K d\log T \end{aligned} \tag{c7}$$

Note that this expression is consistent with a Cobb-Douglas production function since, for that function, the two first right-hand side terms are integrals of constant cost shares and collapse into $\log T$ and $\log K$. Moreover, we have $R(K,T) = \frac{1-\alpha}{\alpha}\, r\, K$ (where α is the share of capital), which implies that $\frac{\partial R(K,T)}{\partial T} = 0$ and the third right-hand side term of (c7) is then zero.